This report was financed predominantly by the « STrategic European Multimodal Modelling (STEMM) » Project [as part of Working Package 4A : « Software design and implementation-Passengers »], Contract No. : ST-96-SC.301 of the Commission of the European Communities, without forgetting MKmetric's cost share of its funding by the Commission. Background support was provided by Marc Gaudry's tenure as 1995-1997 Ernst Blickle Research Prize awardee at *Universität Karlsruhe* and as 1998 *Centre National de la Recherche Scientifique* (CNRS) researcher at BETA, *Université Louis Pasteur* and UMR CNRS 7522; it was also provided by the Industrial Engineering Committee of the National Sciences and Engineering Research Council (**NSERC/CRSNG**) of Canada. The authors thank Christian Calzada of the *Service Économique et Statistique* (SES) of the *Ministère de l'Équipement, du Logement, des Transports et du Tourisme* (MELTT) of France for his exceptional collaboration and the SES for making the INSEE data available.

Methodological Developments within the Quasi-Direct Format Demand Structure : the Multicountry Application for Passengers MAP-1

by

Marc Gaudry [1,2,3]
Florian Heinitz [4]
Jörg Last [2,5]
Benedikt Mandel [1,5]

1 Agora Jules Dupuit (AJD)
Centre de recherche sur les transports (CRT), Montréal
<http://www.crt.umontreal.ca/crt/AgoraJulesDupuit/>
2 Bureau d'économie théorique et appliquée (BETA)
Université Louis Pasteur
Strasbourg
3 Département de sciences économiques
Université de Montréal, Montréal
<gaudry@crt.umontreal.ca>
4 Institut für Wirtschaftspolitik und Wirtschaftsforschung (IWW)
Universität Karlsruhe, <heinitz@iww.uni-karlsruhe.de>
5 MKmetric Gesellschaft für Systemplanung mbH (MKm),
Karlsruhe <mandel@mkm.de>, <last@mkm.de>

Paper presented at the Day Conference on the STEMM Passenger Model organized by the *Service Économique et Statistique* (SES) of the *Ministère de l'Équipement, du Logement, des Transports et du Tourisme* (MELTT) in Paris, France, on October 27, 1998, and at the Second Seminar on National Transport Models organized by PTRC Education & Research Services and Transek AB in Stockholm, Sweden, on December 2, 1998.

Bureau d'économie théorique et appliquée—Working Paper N° 9815
Centre de recherche sur les transports—Publication CRT-98-46

August 1998, Revised April 11, 1999, July 14, 2000, November 11, 2004.

Abstract • Résumé • Zusammenfassung

Abstract

In this paper, we perform four tasks. FIRSTLY, we summarise the seminal fixed-form formulations of coupled two-stage models initiated 30 years ago in the context of Northeast Corridor Transportation Project of the U. S. Department of Transportation and show how the use of Box-Cox transformations made it possible a decade later both to unify the various model formulations and to challenge the credibility of model results obtained under fixed-form component assumptions, notably in gravity-type models of levels and in classical linear Logit-type choice models. SECONDLY, we note that, as soon as either the Level or the Split component of two-stage models are considered separately and can be effected either with aggregate or with disaggregate data, the four possible combinations define a structure, the Quasi-Direct Format, with four matching classes of algorithms.

THIRDLY, we summarise particular streams of improvement of three of these classes, all found in the TRIO program, Version 2.0, and concerned with the generalisation of so-called « root » models in different dimensions : in models of levels, ordinary least-squares procedures are generalised with respect to form of the fixed part and to heteroskedasticity and autocorrelation of the residuals ; in split models, the usual linear Logit model is generalised to non-linear forms that enrich the utility function and solve endemic parameter underidentification problems, and to the addition of choice captivity or modeler ignorance measures. We distinguish between background baseline pre-1996 developments and improvements under STEMM since 1996 : in models of levels, the algorithmic generalisations allow for tests of spatial autocorrelation of residuals with large samples ; in models of splits, numerous extensions of the Logit choice model are made possible with disaggregate data. FOURTHLY, we describe the first-ever multicountry application for passengers of the QDF format, obtained using the TRIO baseline algorithms : MAP-1 consists in a combination of disaggregate mode choice models and aggregate total demand models pertaining to three trip purposes. We also perform advanced tests of the algorithmic improvements effected under STEMM and demonstrate their usefulness on the business trip components of the MAP-1 multicountry application within the QDF structure.

Key-words : travel demand, mode choice, Northeast Corridor, Germany, Europe, INSEE, International passenger Survey, Civil Aviation Authority, regression analysis, Box-Cox, Box-Tukey, Inverse Power Transformation, asymmetry of response, Logit, Dogit, captivity, ignorance, fidelity, heteroskedasticity, serial autocorrelation, directed autocorrelation, spatial autocorrelation, TRIO program, elasticities, probability points, parameter invariance, maximum likelihood, MAP-1 model.

Résumé

Nous réalisons quatre tâches dans cet article. EN PREMIER LIEU, nous résumons les formulations à forme fixe des modèles à deux étapes couplées élaborés il y a une trentaine d'années dans le cadre des études de la demande de transport dans le couloir Nord-Est des Etats-Unis sous l'égide du ministère américain des transports et nous rappelons qu'une dizaine d'années plus tard il a été possible, grâce aux transformations de Box et Cox, d'en unifier les formulations tout en mettant en doute leur crédibilité dans la mesure où leurs résultats avaient été obtenus en supposant pour leurs composantes des formes fixes, notamment la forme gravitaire pour les modèles de niveaux et linéaire pour la composante de répartition de type Logit classique. Nous faisons remarquer EN SECOND LIEU que dès qu'on distingue chaque composante de l'autre et que le modèle de niveaux et le modèle de répartition peuvent chacun être construits avec des données agrégées ou désagrégées, la structure ou Format Quasi-Direct qui en résulte définit quatre combinaisons possibles auxquelles correspondent quatre classes d'algorithmes.

Nous résumons EN TROISIÈME LIEU, pour trois de ces classes, présentes dans le progiciel TRIO en Version 2.0, des directions d'amélioration axées sur la généralisation de « modèles-racines » dans certaines dimensions. Dans les modèles de niveaux, la généralisation des moindres carrés ordinaires se fait par l'introduction de formes souples sur la partie dite fixe du modèle et la prise en compte de l'hétéroscédasticité et de l'autocorrélation des erreurs. Dans le modèle Logit de choix, elle se fait par l'usage de formes non-linéaires qui permettent l'enrichissement des fonctions d'utilité et résolvent les problèmes inhérents d'identifification des paramètres, ainsi que par l'ajout de paramètres qui mesurent la captivité aux alternatives ou l'ignorance de l'analyste. Nous distinguons entre les améliorations antérieures à 1996 et celles qui, depuis, sont imputables à STEMM: il s'agit, pour les modèles de niveaux, d'améliorations algorithmiques autorisant l'usage d'échantillons plus importants pour l'étude de la corrélation spatiale des résidus ; dans le cas des modèles de choix, nous avons programmé plusieurs généralisations du modèle Logit pour les données désagrégées. Nous décrivons EN QUATRIÈME LIEU la première application multinationale aux voyageurs d'une structure QDF en utilisant les algorithmes disponibles dans TRIO : le modèle MAP-1 combine, pour chacun de trois motifs de déplacement envisagés, un modèle désagrégé de choix modal et un modèle agrégé de demande totale. Nous démontrons par ailleurs la pertinence des améliorations algorithmiques occasionnées par STEMM par des essais sur la partie « affaires » de l'application multinationale MAP- 1 sertie dans une structure QDF.

Mots-clés : demande voyageurs, choix du mode, Couloir Nord-Est, Allemagne, Europe, INSEE, Enquête IPS, CAA, analyse de régression, Box-Cox, Box-Tukey, transformation puissance inversée, IPT, réaction asymétrique, Logit, Dogit, captivité, ignorance, fidélité, hétéroscédasticité, autocorrélation de séries, autocorélation dirigée, autocorrélation spatiale, progiciel TRIO, élasticités, points de probabilité, invariance des paramètres, maximum de vraisemblace, modèle MAP-1.

Zusammenfassung

In diesem Artikel werden vier Aufgabenstellungen bearbeitet. ERSTENS, wir fassen die ursprüngliche Formulierung der gekoppelten statischen Zwei-Stufen Modelle zusammen, die vor 30 Jahren im Zusammenhang mit dem Nordost-Korridor-Verkehrsprojekt des Verkehrsministeriums der Vereinigten Staaten von Amerika entwickelt wurden, und zeigen, wie später unter Verwendung der Box-Cox Transformation eine Vereinheitlichung der Modellformulierungen herbeigeführt sowie die Zuverlässigkeit der Ergebnisse dieser auf statischen Annahmen beruhender Modelle erhöht wurde, was die Niveaumodelle ('Level') auf Basis des Gravitationsansatzes als auch die klassischen linearen Logit Wahlmodelle ('Split') umfaßt. ZWEITENS, es wird herausgearbeitet, daß sich bei einer separaten Betrachtung der Niveau- und Wahlkomponente des Zwei-Stufen Modells durch die potentielle Verwendung von aggregierten und disaggregierten Daten vier Fälle ergeben, auch Quasi-Direktes-Format (QDF) genannt, die mittels vier verschiedener Algorithmen zu behandeln sind.

DRITTENS, wir zeigen - unter Verwendung der TRIO Software Version 2.0 - für drei dieser Fälle ausgehend von den generalisierten Ursprungsmodellen Verbesserungsmöglichkeiten unterschiedlicher Dimension auf. Für die Niveaumodelle wurde die Standardform der Methode der kleinsten Quadrate unter Berücksichtigung des systematischen Funktionsbestandteils sowie der Heteroskedastizität und Autokorrelation der Residuen generalisiert. Für die Wahlmodelle wurde die Standardform des linearen Logitansatzes um die Abbildung von nichtlinearen Funktionsformen erweitert, was eine Anreicherung der Nutzenfunktion darstellt und die existierenden Probleme der Unteridentifikation von Parametern löst sowie die Gebundenheit an zur Wahl stehender Alternativen oder des Modellierers Ignoranz quantifiziert. Wir unterscheiden dabei grundlegende Entwicklungen, die vor dem Jahr 1996 stattfanden, und jenen Verbesserungen, die im Rahmen des STEMM Projektes seit 1996 erarbeitet wurden. Bei den Niveaumodellen betrifft dies die Generalisierung der Algorithmen, die nunmehr die Prüfung der räumlichen Autokorrelation der Residuen bei großen Stichproben zuläßt; bei den Wahlmodellen umfaßt dies verschiedene Erweiterungen des Logitansatzes für disaggregierte Daten. VIERTENS, wir beschreiben die erste multinationale Anwendung des quasi-direkten-Formates (QDF) für den Personenfernverkehr, die unter Verwendung der in TRIO implementierten Algorithmen entwickelt wurde. Diese MAP-1 Modelle bestehen für jeden der drei Reisezwecke aus einer Kombination von disaggregiertem Wahlmodell und aggregiertem Nachfragemodell. Es wurden ferner weitreichende Tests der im Rahmen des Projektes STEMM verbesserten Algorithmen durchgeführt, die deren Nützlichkeit anhand des MAP-1 Geschäftsreisendenmodells demonstrieren.

Schlüsselworte: Reisenachfrage, Verkehrsträgerwahl, Nordost-Korridor, Bundesrepublik Deutschland, Europa, INSEE, internationale Passagierbefragung, zivile Luftfahrtbehörde, Regressionsanalyse, Box-Cox, Box-Tukey, Inverse Power Transformation, Asymmetrie der Nachfrage, Logit, Dogit, Gebundenheit an eine Alternative, Ignoranz des Modellierers, Genauigkeit, Heteroskedastizität, serielle Autokorrelation, direkte Autokorrelation, räumliche Autokorrelation, TRIO Programm, Elastizitäten, Wahrscheinlichkeiten, Invarianz der Parameter, Maximum Likelihood, MAP-1 Modell.

Table of Contents

List of figures

List of tables

1. Two-stage transportation demand modelling as a framework[1]

We need a convenient framework to discuss the importance of functional form considerations in regression models, particularly those used in transport demand analysis, and provide a background for the presentation of advances in this area and demonstrations of their relevance. Two-stage models provide this useful framework because their key distinct components, models of <u>levels</u> and <u>probabilistic or share</u> models, cover much of the practical regression field in Economics and Transportation.

<u>1.1 Making total demand responsive to transport network conditions</u>

Until the early 1960’s, it was the practice to explain modal trip demand between any two points by breaking up the problem among at least 3 different steps : if we neglect the problem of service or path choice by mode, the steps involved the successive explanation of (i) total trips produced (T_i) or attracted (T_j) at a point, (ii) the total interzonal flow (T_{ij}) and (iii) the demand by mode ($T_{ij\ m}$). Although these steps were linked in the sense that the result from each became an input to the next one, they « could not reflect the fact that improved transportation performance by one mode or the introduction of a new mode would induce increased total demand for transportation between the two points » (Crow *et al*., 1971). Two streams of models arose in order to obtain more reasonable overall properties.

A. Fixed-form single-stage or direct models : 1963-1969

<u>Utility is not separable</u>. One-step models attempted to explain the demand by mode (T_{ijm}) directly, that is by writing a single equation where appear, denoted by A_i and A_j, various activities at the origin or destination (usually functions of Population and Employment or Output) and the U_{ijm}, or Generalized costs of the modes (usually functions of modal characteristics such as fares, travel time or frequency, and sometimes of socio-economic factors like Income) :

$$T_{ijm} = f\,(A_i, A_j, U_{ij1}, \ldots, U_{ijM}) \quad , m = 1, ..., M \qquad (1)$$

These models, such as Kraft (1963), were in the spirit of microeconomic demand models, attempting to make the demand for one good depend on the prices of all goods, or at least of prices of close substitutes. A branch developed to forecast the demand for « new modes » and included the famous Quandt and Baumol (1966) « abstract mode » model and its successors (Young, 1969), obtained by imposing « genericity » constraints on the money and time coefficients of the modes.

<u>Generalized costs and mode-abstractness</u>. Two specificities of transportation had to be incorporated. Firstly, the fact that money costs (prices) must be complemented by time costs (service levels) to obtain credible results, a requirement that often increased colinearity and probably also covered (Dagenais and Gaudry, 1986) still unresolved specification issues related to time variables, forced many modelers to use restrictions on signs in estimation procedures. Secondly, the resilience of « mode specificity », easily established by relaxing the genericity coefficient constraints embodying « abstractness ».

B. Fixed-form two-stage or quasi-direct models : 1968-1971

<u>Breaking up the problem—in effect a 3-component structure</u>. The second stream formulated two-step three-component models, namely products of two coupled models where the first one explains the total

[1] The first chapter is closely related to Gaudry (1999).

demand for all modes but includes a measure of the utility of all modes, U_{ij}, and the second one « splits » this demand with a mode choice model. The coupling arises from the fact that the overall utility measure is the denominator of the mode split model :

$$T_{ijm} = T_{ij} \bullet p_{ij_m}. \tag{2}$$

or, more explicitly :

$$T_{ijm} = f(A_i, A_j, U_{ij}) \bullet \left\{ U_{ijm} \Big/ \sum_m U_{ijm} \right\} \quad , m = 1, ..., M \tag{3}$$

with

$$U_{ij} = \sum_m U_{ij_m}. \tag{4}$$

These models rapidly produced results that were reasonable without resorting to constraints on regression signs. They were estimated in two separate steps for practical reasons, a situation that has not changed due to computing costs : the only known exception is Laferrière's (1988, 1998) demonstration of statistical gains from joint estimation in his quasi-direct model of the demand for air travel by itinerary, where the total demand for air travel is multiplied by an itinerary choice model.

Multiplicative throughout. The series of two-step models that starts with McLynn *et al*. (1968), used multiplicative forms in all functions of (3). Dropping *ij* subscripts, neglecting both constants and errors, these models can be written less abstractly if we denote any activity variable by A_a, any socio-economic variable by S_s and any modal network characteristic by N_n :

$$T = \prod_a A_a^{\beta_a} U^{\beta_U} \quad a = 1, ..., F \tag{5}$$

$$U = \sum_m U_m \quad m = 1, ..., M \tag{6}$$

$$U_m = \prod_n N_n^{\beta_{mn}} \prod_s S_s^{\beta_{ms}}. \quad n = 1, ..., G \tag{7}$$

In spirit and form, these models, such as those by McLynn and Woronka (1969) are quite close to the contemporaneous quasi-direct multiplicative model written by Armington (1969) to explain international trade flows : the trade flow literature had to specify forms in which each commodity (like a modal flow) was traded everywhere and in both directions (in spite of the theory of comparative advantage) in quantities that depended on all individual commodity prices. To achieve these ends, the trade literature used *a priori* restrictions on the coefficients (written as elasticities of substitution) and extremely simple specifications, typically collapsing all characteristics of goods into a single variable (price) in (7) and the whole structure of the economy into a single activity (GNP) in (5). A recent application of such a quasi-direct multiplicative trade model to the demand for air travel—requiring the inclusion of other network variables than price—is that by Gillen *et al*. (1998).

The linear Logit as a component. The only exception to the use of a multiplicative form within a recognisable quasi-direct model is Blackburn who, in the most tractable version of his model (1969), made use of a linear-in-variables exponential form of (7), effectively writing a linear Logit model :

$$U_m = exp(V_m) \tag{8}$$

$$V_m = \beta_{m0} + \sum_n \beta_{mn} N_{mn} + \sum_s \beta_{ms} S_s \,. \qquad (9)$$

This classical linear Logit model had been used first by Warner (1962) as a binary mode choice model using individual (discrete or disaggregate) observations, studied by Cox (1970) and Ellis and Rassam (1970) and first linked to utility theory by Rassam *et al.* (1970, 1971) in a multinomial application with share (aggregate) data, a linkage later completed by Domencich and McFadden (1975). The model can also be obtained from other random processes : as a by-product of Wilson's (1967) derivation of the Gravity model as most likely arrangement under a total cost constraint ; from the asymptotic theory of extremes (Leonardi, 1982) ; by rearrangement of Theil's « rational random demand » (1975)—as pointed out by Truong (1981)—or for that matter from any process where maximisation involves taking the derivative of natural logarithms, such as entropy maximisation or information minimisation !

We will limit our discussion to the multinomial Logit model and neglect more complicated nested constructs because they are not central to our purpose here. Also, applications of the disaggregate Logit model to frequency and destination choice were late in coming and much more difficult than applications to the choice of mode : in that sense, the Logit model has become the workhorse of mode choice analysis but has yet to demonstrate its advantages over aggregate models in generation-distribution models—were that ever to be the case, it will be our implicit point that such a demonstration would not be credible without due consideration of functional forms.

C. Unification of quasi-direct models—the power of flexible forms : 1976-1981

Nesting previous models in a general form. These various models were compared as nested special cases of a quasi-direct model by Gaudry and Wills (1977, 1978) through the use of Box-Tukey and Box-Cox (1964) direct power transformations, defined with parameters λ_κ and μ_k on the positive variable X_k as in Table 1.

Table 1. Transformations defined by Box, Cox and Gaudry

Ref.	Box and Cox (1964)		Gaudry (1981)		
Def.	Direct Power Transformation (DPT) of X_k		Inverse Power Transformation (IPT) of Z_k		
Code	$X_k(\lambda_k, \mu_k)$	Condition	$Z_k(\phi_k, \mu_k)^{-1}$	Conditions	Code
BC	$\frac{X_k^{\lambda_k} - 1}{\lambda_k}$	$\lambda_k \neq 0,\ X_k \rangle 0$	$(\phi_k Z_k + 1)^{1/\phi_k}$	$\phi_k \neq 0,\ (\phi_k Z_k + 1 \rangle 0)$	BCG
	$ln\ X_k$	$\lambda_k = 0,\ X_k \rangle 0$	$exp(Z_k)$	$\phi_k = 0,\ (Z_k \rangle 0)$	
BT	$\frac{(X_k + \mu_k)^{\lambda_k} - 1}{\lambda_k}$	$\lambda_k \neq 0,\ (X_k + \mu_k \rangle 0)$	$(\phi_k Z_k + 1)^{1/\phi_k} - \mu_k$	$\phi_k \neq 0,\ (\phi_k Z_k + 1 \rangle 0)$	BTG
	$ln(X_k + \mu_k)$	$\lambda_k = 0,\ (X_k + \mu_k \rangle 0)$	$exp(Z_k) - \mu_k$	$\phi_k = 0,\ (Z_k \rangle 0)$	

Although both the BC and BT forms are found in that 1964 paper, the expression « Box-Cox transformation » had come to denote only the BC form in common parlance, because Box and Cox had not found the shift parameter μ_k to be statistically significant : this limited its role to that of insuring that the variable considered becomes strictly positive before being raised to the power λ_κ . The absence of any other interpretation of this shift parameter also contributed to its falling into disuse and to the BC form progressively becoming the most widely used monotonic transformation of variables in applied work (Davidson and MacKinnon, 1993). The Gaudry and Wills paper demonstrated that the shift parameter, reintroduced under the name « Box-Tukey », could be statistically significant when applied to *variables* of the 4 time-series and cross-sectional models considered. The paper did not yet provide for the Tukey shift parameter the new behavioural interpretation that was to come later (Gaudry, 1981) by considering the BCG and BTG inverse forms and applying them to complete *functions* U_m or V_m in (8). This application was conceived from the realization that one could not directly transform the dependent variable of a Logit model and easily maintain the requirement that the choice probabilities sum to one, but that one could achieve this result indirectly by using inverse transformations on the right-hand side U_m functions of this model. However, the paper showed in a number of ways the decisive practical importance of Box-Cox transformations in terms of model properties, data fit, parameter signs and values, to be summarised and elaborated on presently. Some questions pertaining to the numerical and statistical costs of form estimation will be addressed in a later section.

Model specification and meaning. Quite naturally, using BC on variables will change the meaning or properties of models of all types. Interesting differences arise with respect to common practice.

« Models of levels ». In Economics, the BC had been used previously in « models of levels » — where the dependent variable y is strictly positive but otherwise unrestricted—precisely because it includes both linear ($\lambda_y = 1$) and logarithmic ($\lambda_y = 0$) cases in nested fashion, and one is often interested in testing whether the effects are additive, multiplicative or other in the following, where the observation indices are introduced and denoted by *t :*

$$y_t^{(\lambda_y)} = \sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + u_t \quad . \tag{10}$$

In our case, there is a particular interest in the utility term U in (5) because the index (11-A)

$$\left.\begin{aligned} &\text{(A)} \quad U^{(\lambda_U)} = \frac{(\sum_m e^{V_m})^{\lambda_U} - 1}{\lambda_U} \quad , \\ &\text{(B)} \quad V_m = \beta_{m0} + \sum_n \beta_{mn} X_n^{(\lambda_{X_n})} + \sum_s \beta_{ms} X_s^{(\lambda_{X_s})} \end{aligned}\right\} \tag{11}$$

yields the Williams-McFadden (1977) log sum term as a special case if the Box-Cox transformation is such that $\lambda_U \rightarrow 0$ in (10) :

$$U^{\beta_U} \equiv \left\{ \beta_U \ ln\,(\sum_m e^{V_m}) \right\} . \tag{12}$$

As a matter of fact, the results obtained for 1972 intercity Canadian flows were extremely close to the natural logarithm of the U variable constructed from 4 modal utility functions with 3 network variables, each having its own λ_k (and also sharing a μ_k not shown in (11-B)) ; for the other activity variables, they were different from both 0 and 1 and amply justified the general specification (10). This demonstration was useful because of the prevalence, in transportation, of multiplicative forms for generation-distribution models. Kau and Sirmans (1979) worked at the same time on a more limited application to a gravity model. In split models, the term (12) becomes the key coupling mechanism in the hierarchical logit model and advanced models even weigh its components (Mandel, 1998).

Outside of transportation, *a priori* multiplicative forms continue to be used without further testing, even in areas like trade modelling where gravity models (with a *U* term reduced to a simplistic distance proxy measure) are prevalent : Gaudry *et al*. (1996) appear to provide the first challenge to these practices, using intra-European and intra-North American trade flows.

« Probabilistic and Share models ». The BC had not been used previously in Logit models. The so-called « standard Box-Cox Logit » used in the Gaudry and Wills paper was shown to be a powerful general form naturally including both the standard linear Logit model and the multiplicative form (7) of the classical marketing and of the mode split literatures, where it was used simply or with additional restrictions on the β_{mn} in many quasi-direct models. In addition to this classification role, the « standard Box-Cox Logit » also possesses more reasonable properties than the popular linear form. Indeed, the non-linear Box-Cox form (i) makes the effect of a network improvement depend on the level of the characteristic, as shown in Table 2 : this means that the impact of a 10 minute change in travel time is not the same for a short and for a long trip...; (ii) makes derived marginal rates of substitution between time and money (values of time) vary both across modes (due at least to different sample levels of the characteristics) and with the amount of time saved. The standard Box-Cox Logit therefore avoids much market segmentation used to obtain reasonable and variable trade-offs by distance, income, etc.

Table 2. Non constant returns in a Box-Cox Logit model

	$\frac{\partial U_m}{\partial X_{mk}} = \beta_{mk} X_{mk}^{(\lambda_{mk}-1)}$	$\frac{\partial U_m^2}{\partial X_{mk}^2} = \beta_{mk}(\lambda_{mk}-1) X_{mk}^{(\lambda_{mk}-2)}$	Returns
$\lambda = -1$	β_{mk} / X_{mk}^2	$-2\beta_{mk} / X_{mk}^3$	Decreasing
$\lambda = 0$	β_{mk} / X_{mk}	β_{mk} / X_{mk}^2	Decreasing
$\lambda = 1$	β_{mk}	0	Constant
$\lambda = 2$	$\beta_{mk} X_{mk}$	β_{mk}	Increasing

In retrospect, it is amazing that a model with constant marginal effects and additive utility would establish itself as a reference model anywhere and come to supersede more credible non-additive utility forms, e. g. the multiplicative one, without notice being taken of test results—that invariably lead to rejection of the linear Logit in favor of a form like the square root or the multiplicative, as one would expect on the basis of classical utility theory. Indeed, in passenger studies, Koppelman (1980), Hensher and Johnson (1981), Mandel (1992, 1999) and Mandel *et al*. (1997) have clearly shown the superiority of the Box-Cox Logit over the linear Logit in passenger studies. Applications to freight are extremely recent (Fridström and Madslien,1995 ; Picard and Gaudry, 1998).

If more reasonable properties and meanings cannot change the habit of presenting models of untested form, what of other motivations concerning data fit and elasticities ?

Model fit and results in general. By definition, the relaxation of constraints in a model must improve the fit and likelihood ratio tests can be used to determine how significant these gains are. The best way to understand model fit is to graph the curves that are drawn by the estimation procedures as these can always be thought of as summarising the information by « fitting » a curve of some shape through the data. This is true in all models and can easily be illustrated in graphs showing the value of the dependent variable y against an explanatory variable X_k. We shall first discuss the issue of form by assuming that a BC is used on the dependent variable y and/or one or more independent variables X_k either in models of levels or in split models of the Logit type. In these cases, it is used as a monotonic transformation. We shall also say a few words about the use of BC to detect quadratic effects.

Form in models of levels. The many models that are special cases of the Box-Cox model of levels[2] are defined in Figure 1 and shown in Figure 2 for these convenient homely forms. The linear form is used because of the convenient statistical properties derivable under linearity assumptions, despite the often heroic assumption that a consequence can be decomposed into linearly additive causes and the bother of calculating elasticities that depend on where the change is considered. The log-log form is used because it implies interaction among causes and constant elasticities. Naturally, the « cross » forms, the log-inverse (exponential) and the semi-log, are less used because they involve more work on the part of the researcher, due to the somewhat more complicated elasticity calculations : they are only worthwhile if the combinatorial game played by the analyst yields desired results only with this form.

There is in general no good reason to expect these pure cases to give the best adjustments to the data : as in Figures 3-4, « true » shapes may be anywhere between the linear and the semi-log case, or between the latter and the log-log case. There are now many sound examples demonstrating that the best fit is indeed often between the linear and log cases in a variety of transportation demand models.

Form in Logit models. It can be pointed out that, as shown in Figure 5, the linear Logit model is symmetric with an inflexion point at probability equal to ½ : by contrast, the presence of non-linearity implies asymmetry of the response curve. Deepack and Laferrière (1994) and Mandel *et al.* (1997) demonstrated, respectively on the Canadian and German rail networks, how much impact this could have on market share gains following a High Speed Rail improvement. Intuitively, a response curve is not likely to be symmetric : a utility function cannot be credible without being demonstrated to be so ! For instance, using an excellent data set on mode choice in Santiago de Chile, Gaudry *et al.* (1989) find that, in linear form, the models has elasticities of in-vehicle time greater than elasticities of wait time, and values of time that fall as incomes rise ! These results correct themselves as soon as a single BC transformation is applied to all variables.

Further unpublished work on these data (Pong, 1991) also showed that the Train-McFadden (1978) goods/leisure trade-off assumed in this model, represented by the ratio of modal cost to income, is easily rejected for a more general specification, where each term is self-standing, for all functional forms except the linear—itself completely dominated by more general BC forms that also imply more reasonable trade-offs (marginal rates of substitution) between time and money, so-called values of time.

Changing form can change regression signs. As just implied by statements about results in general, elasticities will differ from what they would have been had a non-optimal form been used. The 1978 paper showed this with graphs of elasticities changing continuously with the BC parameters. But it also showed what practitioners had long known : changing the functional form can change *not only the size of coefficients, but also their sign* ! In multivariate models, because regressors are not generally orthogonal (uncorrelated) in any form, the sign obtained for a given variable depends on the covariances among regressors—depends on the functional form used. In the paper, this was demonstrated with models of levels but is also true of split models. There are many examples of such sign reversals even in published articles.

[2] Figures 1-4 are minor reeditions of overhead sheets drawn from A. Blanquier's presentation « Using the TRIO Level 1.4 Algorithm at *SETEC-économie», Ecole Nationale des Ponts et Chaussées* (ENPC), Paris, November 25, 1993.

Figure 1. Box-Cox models of levels : four nested classical models

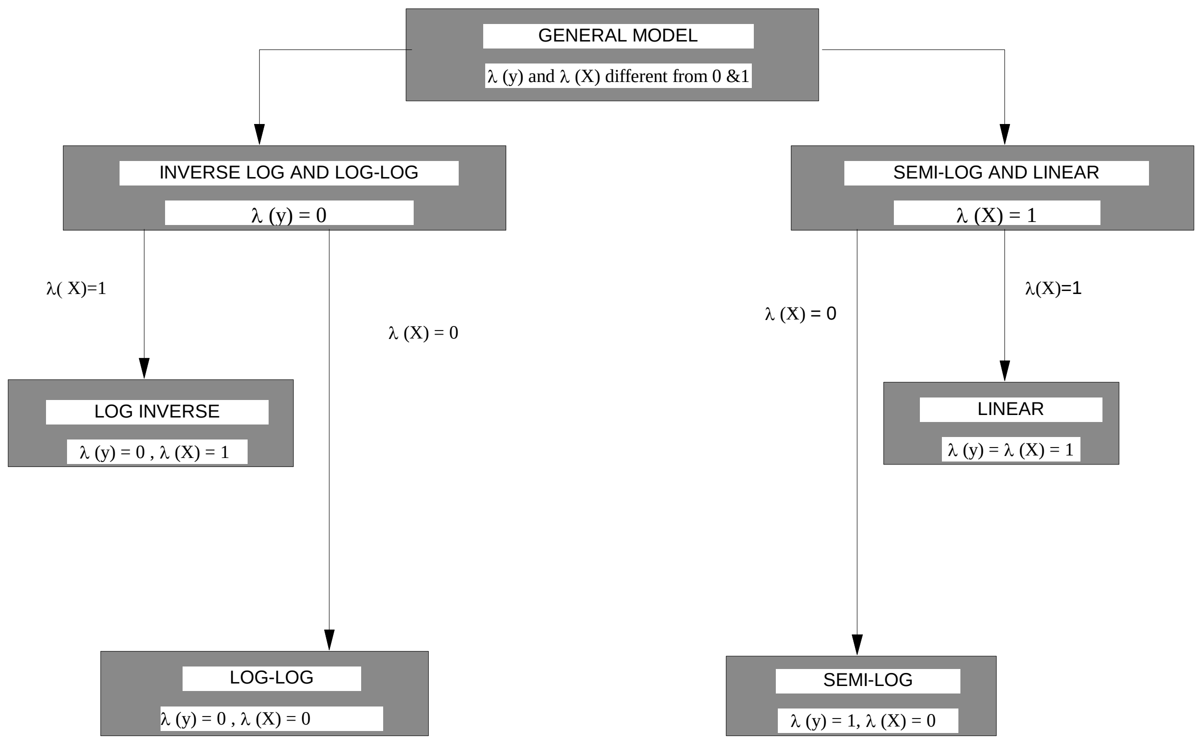


Figure 2. Four classical shapes in linear regression

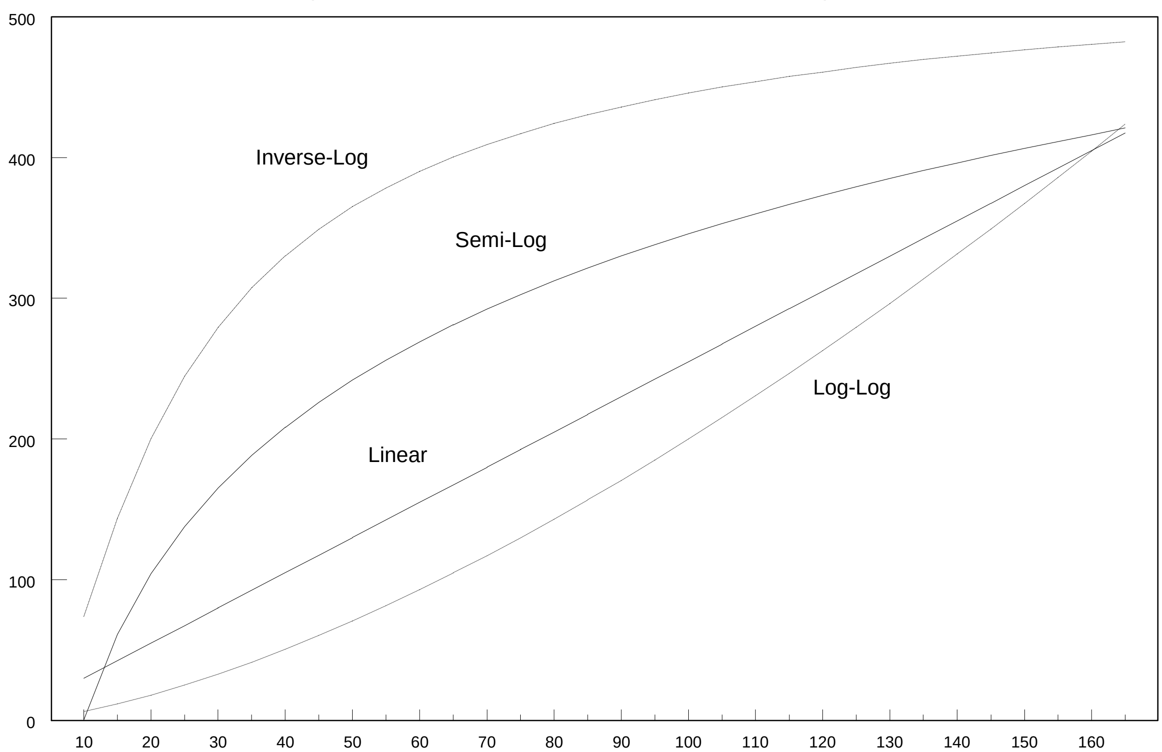


Figure 3. From linear to semi-log form with Box-Cox

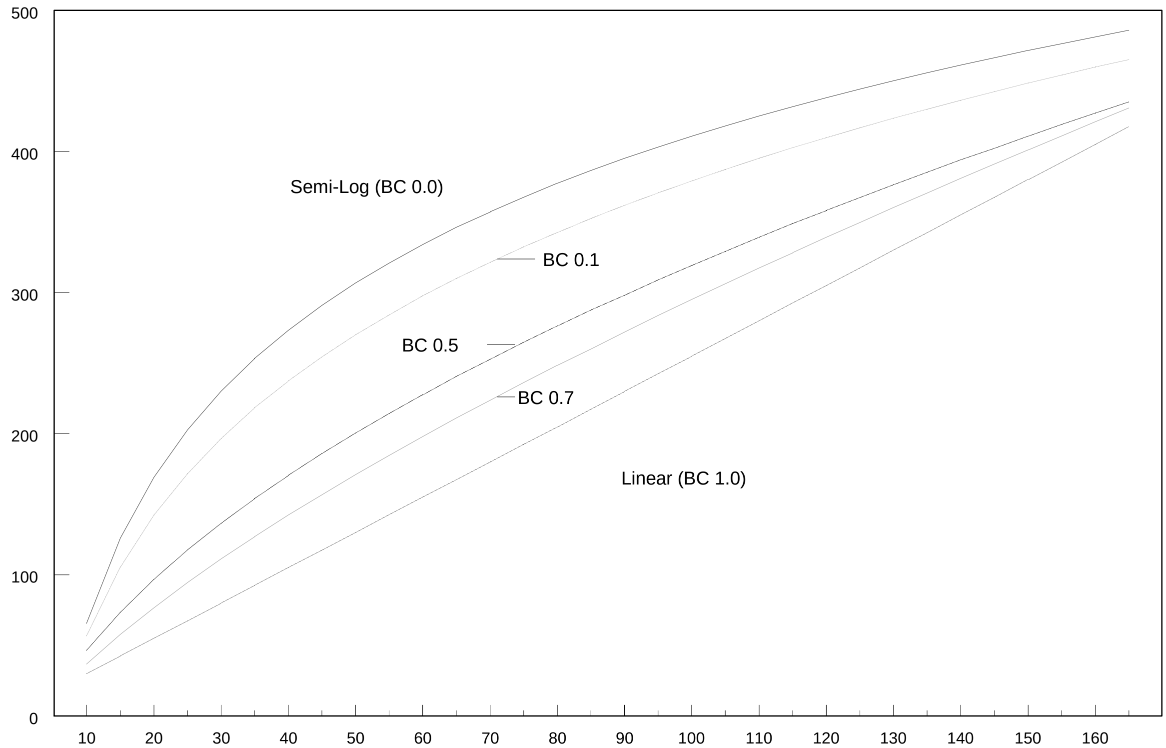


Figure 4. From log-log to semi-log form with Box-Cox

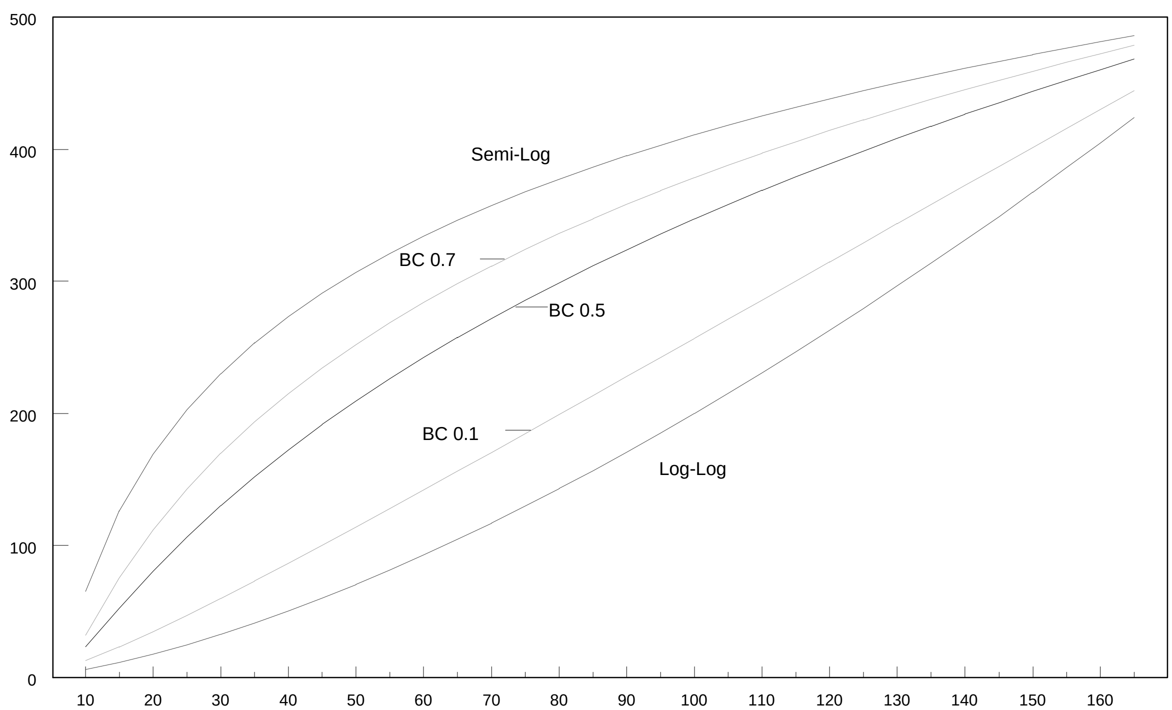


Figure 5. Linear-Logit vs Box-Cox-Logit

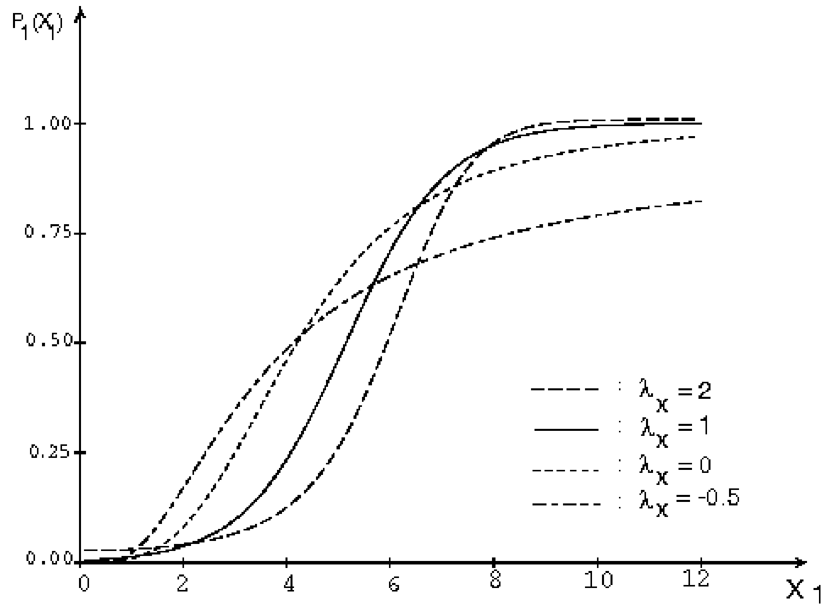


Figure 6. Linear-Logit vs Standard-Dogit

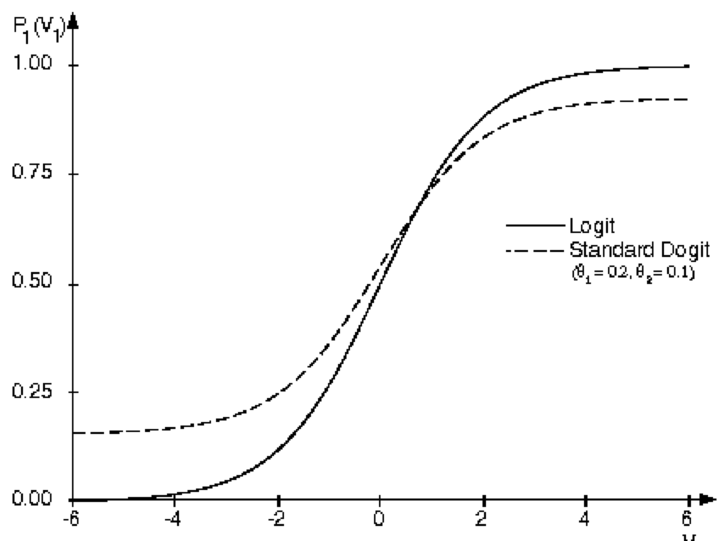


Figure 7. Linear-Logit vs Linear-Inverse-Power-Transformation-Logit

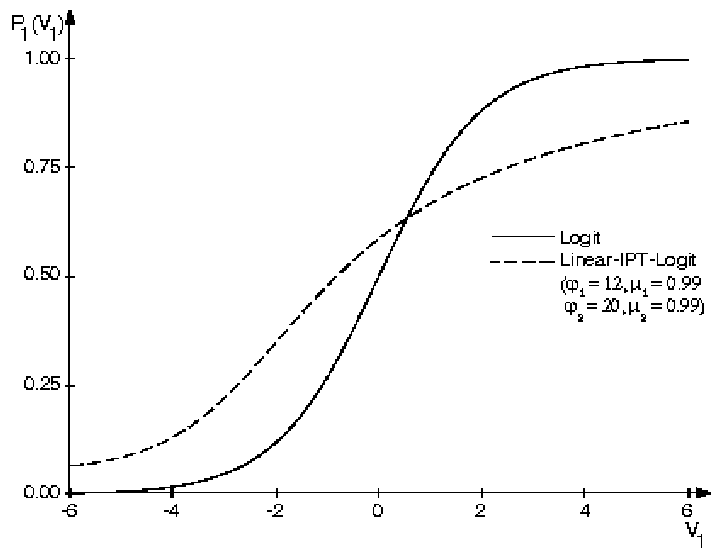
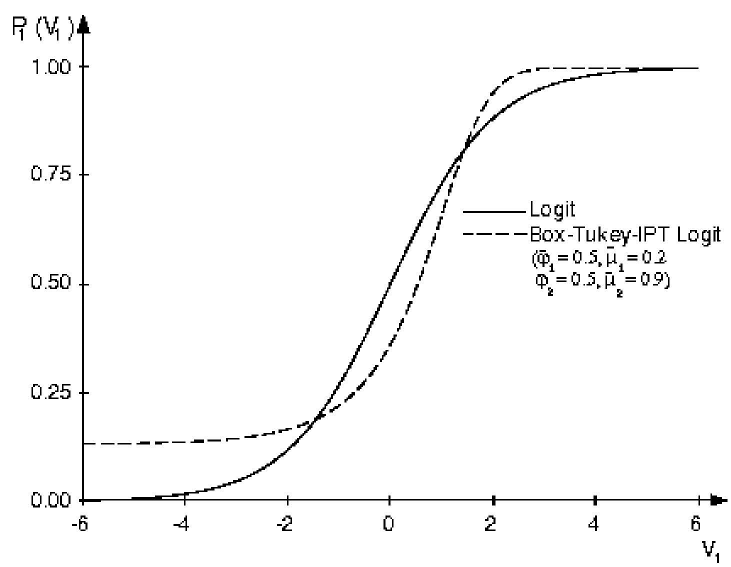


Figure 8. Linear-Logit vs Box-Tukey-Inverse-Power-Transformation-Logit



Signs in « Models of levels ». Despite this clear visual demonstration of signs changing with the functional form used, illustrated in Figure 9.A, many papers have been written during the last 20 years after using a « by hand » search for the form that gives the *desired* results. For instance, in a recent study of the impact of competition on the dispersion of prices charged by airlines, the authors (Borenstein and Rose, 1994) obtain, as shown in Figure 9.B, positive and significant signs for 3 of their crucial variables (pertaining to the degree of competition among airlines) when the model is linear, but negative and significant signs when the model is multiplicative (log-linear); they then accept the linear model without formal tests of the appropriate form : such tests might well invalidate their preferred finding, perhaps by revealing a point between the linear and logarithmic cases, and *insignificant* results , or even a point not too far from the log-linear point...and *contrary to their expectations*! Mind you, the same issues arise outside of transportation as well as in other domains of application : for instance, Blum and Gaudry (1990) demonstrate, as shown in Figure 9.C, that the very form of the savings function largely determines the results obtained and the *direction* of influence of the variables in the discussion on the impact of social security contributions on household savings.

Signs in Logit models. In Logit models, the issue is not just one of sensitivity to form, as shown in Figure 10.A, but also of the robustness of regression signs. A few years ago, Gaudry (1985) reproduced the linear Logit results obtained by R.A.T.P. and Cambridge Systematics (1982) using a first rate data base on Paris work trips (Moïsi *et al*., 1981) and easily showed, as indicated in Figure 10.B, using only one BC transformation that dramatic gains in fit could be achieved with an optimal BC value at mid-point between 0 and 1 but that, at that point, the parking cost variables obtained an « incorrect » sign in the R.A.T.P.'s favourite model that had required two years of work ! More recently, Fridström and Madslien (1995) obtain the correct sign on transport risk in a Logit model as soon as they use BC transformations, as indicated in Figure 10.C.

It is well known that practitioners arbitrarily play with the functional form until they obtain desired results. It is therefore hard to resist this recent « Proposition on Form » :

PF-1 : Models of untested functional form are not credible, irrespective of the economic or other theory on which they are based. Such tests, applied to many current models using predetermined monotonic forms would show that the models are not robust. This is true irrespective of the kind of data used : revealed and stated preference data, experimental data...(Gaudry, 1998)

Asymmetric quadratic effects on form. Also on this point that if, in a model of the level class such as (13), a variable X is used both linearly and with a BC, $\lambda_X = 2$ yields the normal symmetric (integer=2) quadratic form and any other value (except 1) yields an asymmetric U-shaped form (see Figure 11) independently from the value of λ_y, if the two regression coefficients are of opposite signs (with the conditions for a maximum or minimum reversed depending on whether the BC on X is smaller or greater than 1)—a result (Gaudry, 1996), that should also hold *mutatis mutandis* in a Logit model :

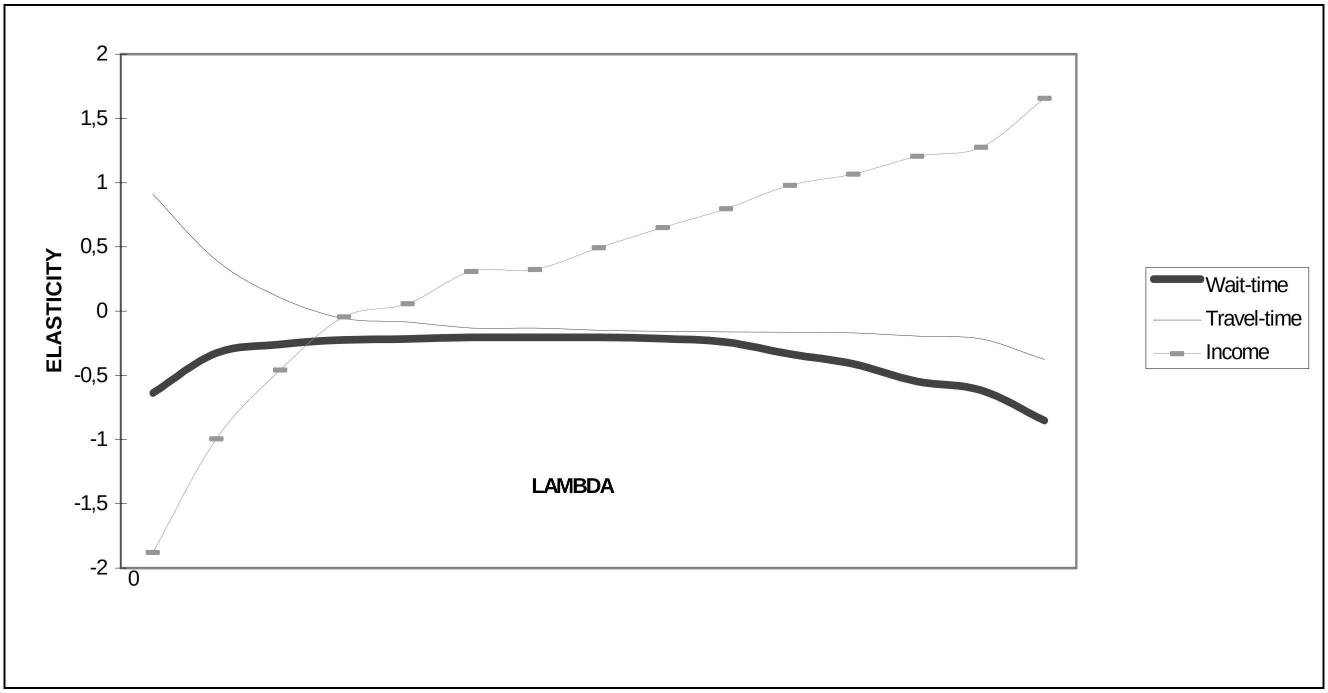
$$y^{(\lambda_y)} = \beta_0 + \beta_1 X + \beta_2 X^{(\lambda_X)} + u. \qquad (13)$$

1.2. From a model to a format : the possible key component data combinations

We pointed out above that two-stage models provide a useful framework because their key distinct components, models of levels and probabilistic or share models, each widely used in its own right, can be discussed separately. We have explicitly neglected to discuss the data types that can be associated with each step and their possible combinations within a common quasi-direct format (QDF).

Figure 9. Form and Signs in Models of Levels

A. Transit Demand in Montreal
Gaudry and Wills, 1978 ; Results from Figure 19
Wait-time, Travel Time and Income Elasticities
y variable : monthly transit demand by schoolchildren



λ goes from 0 to 2 on the X-axis

Comment : note how variables change signs. The optimal value is close to one : at that point, public transit is a superior good and loses clients if in-vehicle time incrases. The opposite occurs with a non-optimal logarithmic form.

B. **Airline Price Dispersion Market Structure, U.S.A.** Borenstein and Rose, 1994 ; Model 2 from Table 3			
Coefficients and t-statistics conditional on form			
y variable :	**Gini ticket price dispersion index**		
X variables...	(4) Linear	(3) Log-log	Optimal λ
[...]			
Monopoly	+0,154	-2,169	n.c.
	(+4,81)	(-5,27)	n.c.
Duopoly	+0,174	-2,033	n.c.
	(+4,97)	(-9,46)	n.c.
Large-duopoly	-0,022	-0,117	n.c.
	(-2,77)	(-0,21)	n.c.
Small-duopoly	-0,017	-0.067	n.c.
	(-1,89)	(-1,10)	n.c.
Competitive	+0,172	-1.807	n.c.
	(+7,16)	(-6,98)	n.c.
Lambda (λ)	1,00 Fixed	0,00 Fixed	n.c.
Log-likelihood	n.c.	n.c.	n.c.

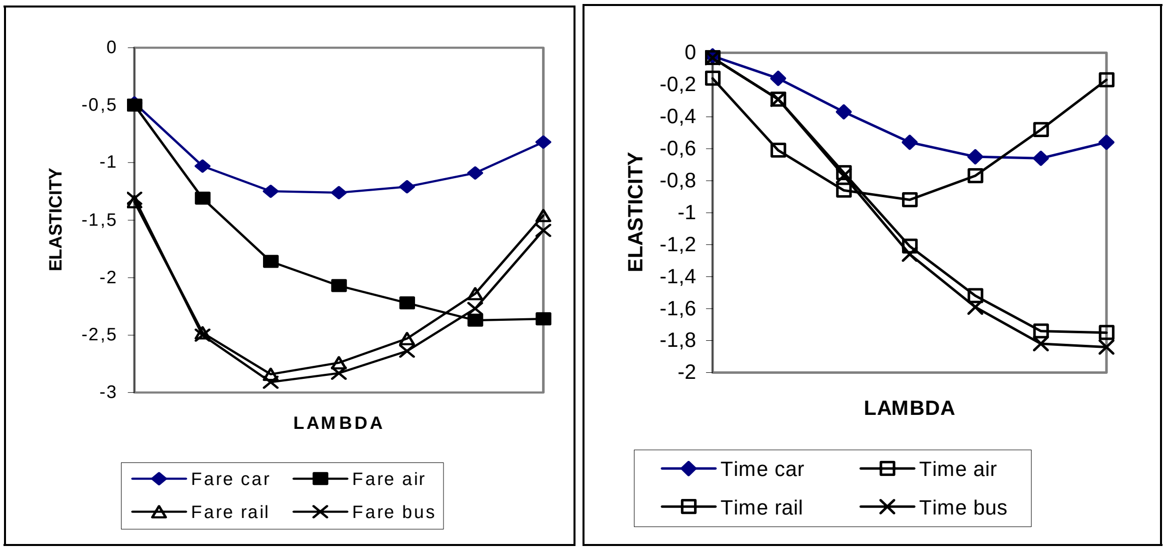
Comment : No log-likelihood values are reported by the authors who arbitrarily choose Model (3) after stating that « the main qualitative results are robust to changes in functional form » (sic !).

C. Household Savings, Germany Blum and Gaudry, 1990 ; Models from Tables 4 and 5			
Elasticities and t-statistics conditional on form			
y variable :	**Annual household savings**		
X variables	(1) Linear	(2) Log-log	(3) Optimal λ
[...]	**a) Self employed**		
Social insur.	-0,149	+0,054	-0,048
contributions	(-10,24)	(+2,86)	(-5,35)
Lambda (λ)	1,00 Fixed	0,00 Fixed	+0,648
Log-likelihood	65,477	-649,585	248,304
[...]	**b) Low-income white-collar**		
Social insur.	-0,962	+0,245	-0,836
contributions	(-2,97)	(+0,28)	(-2,41)
Lambda (λ)	1,00 Fixed	0,00 Fixed	+0,748
Log-likelihood	-139,074	-293,503	-126,687

Comment : The impact of social security contributions on household savings is expected to be negative, as it is in both cases when the optimal form is used. The optimal point is close to half way between the linear and logarithmic.

Figure 10. Form and Signs in Logit Models

A. Intercity Demand in Canada
Gaudry and Wills, 1978 ; Results from Figure 6 and Figure 7
Fare and Travel Time Elasticities
Modal shares : Car, Air, Rail, Bus



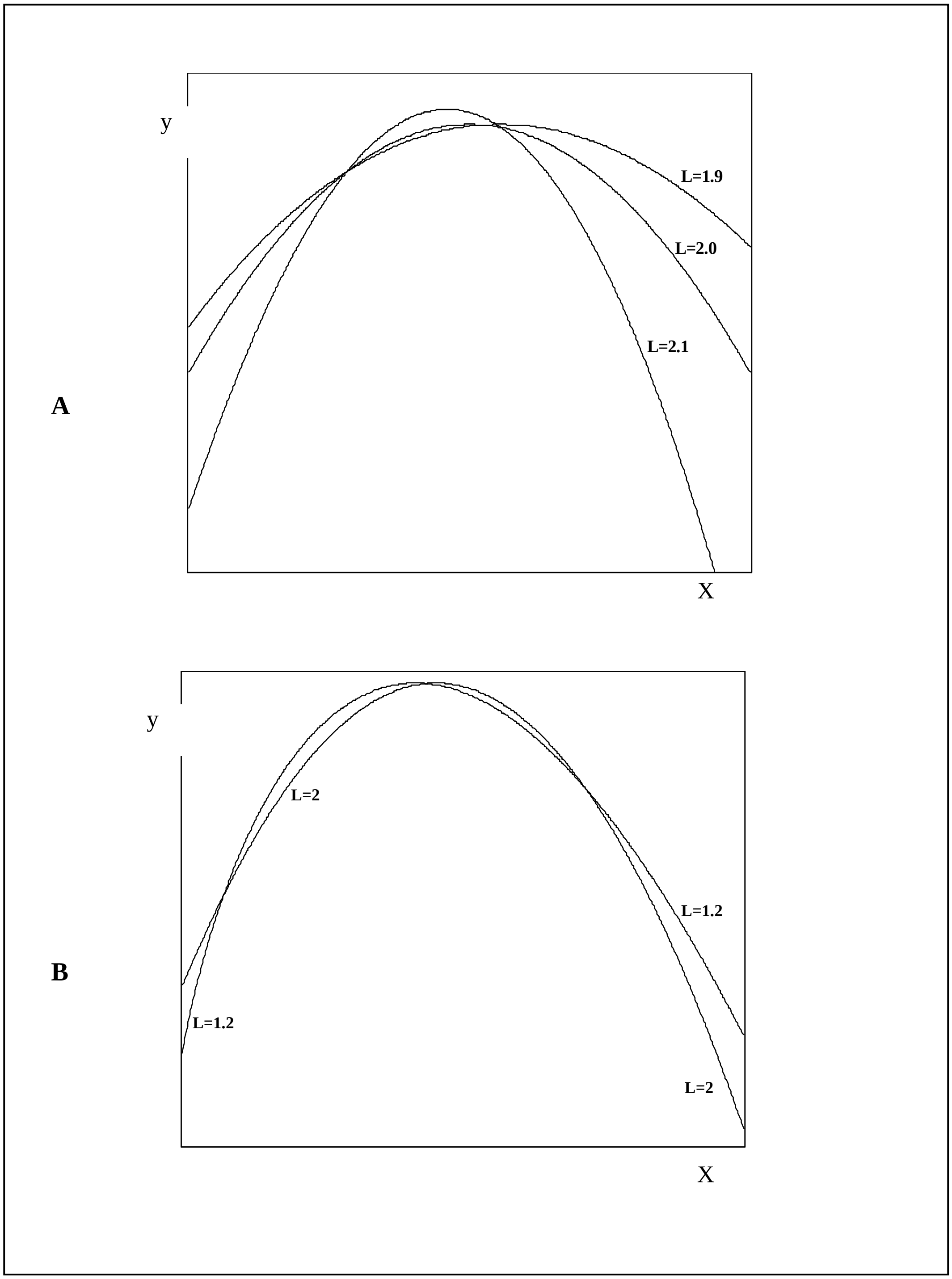
λ goes from -1 to +1 on the X-axis

Comment : Models with different elasticities have different properties.

B. Trip to Work in Paris Gaudry, 1985 ; R.A.T.P. Models from Table 3		
Elasticities and t-statistics conditional on form		
Choice between 6 modes : by Rail, Bus and Rail, Bus, Private Car, Motorized Two-Wheel and on Foot		
X variables...	Linear	Optimal λ
[...]		
Parking cost/inc. socio cat. 1,3	-0,01 (-3,23)	0,00 (+2,76)
Parking cost/inc. Socio cat. 2,4	-0,01 (-2,48)	0,00 (+1,70)
Lambda (λ)	1,00 Fixed	0,500
Log-likelihood	-911,82	-904,03
Comment : none of the other variables changed signs and a single BC is involved		

C. Wholesalers' Freight Shipments in Norway Fridström, L. and A. Madslien, 1995 ; Models A-1 and A-2		
Coefficients and t-statistics conditional on form		
Choice between 4 alternatives : 2 For-Hire and 2 Own-Account Options		
X variables...	Linear	Optimal λ
[...]		
Late delivery risk of a general nature	+0,0036 (+2,50)	-0,043 (-4,24)
[...]	-- --	-- --
Lambda (λ)	1,00 Fixed	0,536
Log-likelihood	-1775,34	-1595,36
Comment : none of the other variables changed signs and 3 BC are involved (for Time and Cost).		

Figure 11. From symmetric ($\lambda_x = 2$) to asymmetric ($\lambda_x \neq 2$) quadratic forms



Four principal data types for each QDF component

To simplify, data belong to two different classes : aggregate [*a*] , or disaggregate [d] or count. Within each class, data may pertain to a cross-section [s] or to a time-series [t]—we neglect pooled data—of observations. If these four types, denoted by a pair of symbols, for instance $\langle a \mid s \rangle$ for aggregate cross-sectional data, are combined within a Quasi-Direct Format model, 16 combinations are possible. However, only a few have been used frequently.

For instance, all Northeast Corridor quasi-direct models of $T_{ij,m}$, trips by mode by origin-destination pair, including Blackburn's model by income class, are aggregate cross-sectional in both product component dimensions of (2):

$$\left[\, {}^{\langle a \,\|\, s \rangle}T_{ij_m} \right] = \left[\, {}^{\langle a \,\|\, s \rangle}T_{ij} \right] \bullet \left[\, {}^{\langle a \,\|\, s \rangle}p_{ij,m} \right] \qquad (13)$$

By contrast, Tegnér *et al*. (1998) formulate a quasi-direct model of $T_{i,\,mop}$, the demand for transit by ticket type (mode of payment, denoted *mop*) in Stockholm with aggregate time-series data:

$$\left[\, {}^{\langle a \,\|\, t \rangle}T_{i,mop} \right] = \left[\, {}^{\langle a \,\|\, t \rangle}T_{i} \right] \bullet \left[\, {}^{\langle a \,\|\, t \rangle}p_{i,mop} \right] \qquad (14)$$

In the Strategic European Multimodal Model (STEMM) Multicountry Application for Passengers (MAP-1) demonstration model below, the trip demand $T_{ij,m}$, is obtained by multiplication of cross-sectional estimates obtained from an aggregate model of total trips and a disaggregate mode choice model :

$$\left[\, {}^{\langle a \,\|\, s \rangle}T_{ij,m} \right] = \left[\, {}^{\langle a \,\|\, s \rangle}T_{ij} \right] \bullet \left[\, {}^{\langle d \,\|\, s \rangle}p_{ij,m} \right] \qquad (15)$$

which requires some care in evaluating the U term used in the coupling mechanism, as will be noted in the discussion of the algorithm appropriate to this mechanism.

B. Model classes and QDF combinations

In effect, as the distinction between time-series and cross-sectional data types is considered to define data types within data classes, the 4 component model classes found in Table 3, the Level, Count, Share and Probability classes, afford 4 different QDF combinations. But we have not found examples of Count-Share and Count-Probability models. We shall therefore focus our attention on Level, Share and Probability algorithms and only marginally refer to Count algorithms.

Table 3. Combining regression model classes in a Quasi-Direct Format (QDF)

Model classes		Models of shares	Models of probabilities
	Relevant data class	*Aggregate*	*Disaggregate or discrete*
Models of levels	*Aggregate*	Level-Share	Level-Probability
Models of counts	*Disaggregate or Count*	Count-Share	Count-Probability
		Quasi-Direct Format combinations	

2. Methodological developments within three principal model classes

We just stated that it was convenient to distinguish among four regression model classes associated with two types of data and that we would focus our attention on three such classes, limiting our comments on count data procedures to a paragraph. To be more precise about regression model classes, it is important to be more precise about what is meant by aggregate and disaggregate data. We shall use for this purpose the TRIO terminology (Gaudry *et al*., 1993-1997) and then describe both the baseline algorithms found in Version 2.0 of the program and the recent developments over the period of the STEMM project (1996-1998), including the development of the QDF utility to combine model results.

<u>2.1. Regression model classes, families and types</u>

A. Defining regression classes, families and types

To gain precision on model classification, consider the following general regression problem :

$$y_m = f_m(X_m; \Pi_m, u_m), \quad , m = 1, . . . , M, \qquad (16)$$

where, for the m[th] equation of interest, $f_m(\cdot)$ is an unspecified function relating the continuous dependent variable y_m to a vector of exogenous variables X_m, Π_m denotes the parameters to be estimated and u_m the error terms. Note also that there are no endogenous right-hand side variables.

With this general problem in mind, one can understand the formal definitions found in Table 4 :

• **classes** are defined by *limitations on the possible values of the* y_m (on their individual range and on whether they must sum up to 1) due to the set of regression functions considered and the (continuous or discrete) *nature of observations* available on the y_m ;
• **families and types** are defined by *limitations on the range of observations* and by *constraints on the specification of the* $f_m(\cdot)$. A particular **family~+~type** specification is defined by considering jointly a systematic part (involving the regressors) and a stochastic part (or model of the residuals) of a model.

B. Algorithmic framework

<u>*Model classes and roots*</u>. The philosophy of the LEVEL, SHARE and PROBABILITY algorithms is to allow the analyst to start with three simple univariate or multivariate models and to build gradually more complex extensions—in arborescent fashion—that contain the simpler ROOT models as nested special cases. Within all 3 classes, these extensions modify the functional form of the functional relationship and allow enrichments of the specification ; within the LEVEL class, they take into account various ways of extracting systematic information from the residual errors in order to obtain a spherical distribution—that is, purely random and of constant variance. The ROOT models are the **Ordinary Least Squares** (OLS) procedure and the classical linear-exponent LOGIT, called LOGIT **(Standard-Linear)**, procedure. These two ROOT procedures are well known and widely used.

<u>*Maximum likelihood estimators*</u>. In all cases, a maximum likelihood procedure is used and documented in full in the reports describing the algorithms. In this summary of program design and development, we shall then concentrate on the formulations and their meaning in order to give the reader a feel for the point of the procedures and their relevance. By covering both background and recent work, we shall in effect give an overview of each of the 12 new model families found in the 3 classes defined in Table 4.

Table 4. TRIO definitions of model class, family and type

	Values of the dependent variable y_m				Specification of the $f_m(\bullet)$					
	in the regression		in the sample		in systematic and stochastic parts					
CLASS	$\sum y = 1$	**range R**	**nature**	**range S**	**Algor. Name**	**ROOT¬ ~~FAMILY**	**TYPE**			
	no	$-\infty \prec y_m \prec \infty$		same as R	OLS	**OLS¬**				
LEVEL	no	$0 \prec y_m \prec \infty$	conti-nuous	same as R	L-1.4	**~BC-GAUHESEQ**	LIN	+BC	+HG	+GAU
					L-2.0	**~BC-DAUHESEQ**	LIN	+BC	+HG	+DAU
					S-1	**LOGIT ¬**	S-LIN		S-BC	G-BC
					S-2	**~ S-DOGIT**	S-LIN		S-BC	G-BC
SHARE	yes	$0 \prec y_m \prec 1$	conti-nuous	same as R	S-3	**~ G-DOGIT**	S-LIN		S-BC	G-BC
					S-4	**~ LIN-IPT-LOGIT**	S-LIN	G-LIN	S-BC	G-BC
					S-5	**~ BT-IPT-LOGIT**	S-LIN	G-LIN	S-BC	G-BC
					P-2	**LOGIT ¬**	S-LIN		S-BC	G-BC
PROBA-BILITY					P-3	**~ S-DOGIT**	S-LIN		S-BC	G-BC
	yes	$0 \prec y_m \prec 1$	discrete	y_m=0 or 1	P-4	**~ G-DOGIT**	S-LIN		S-BC	G-BC
					P-5	**~ LIN-IPT-LOGIT**	S-LIN	G-LIN	S-BC	G-BC
					P-6	**~ BT-IPT-LOGIT**	S-LIN	G-LIN	S-BC	G-BC
	Defines **3** ***CLASSES***				Defines 13 ***FAMILIES***		2 to 4 ***TYPES per family***			

Note to Table 4

Meaning of FAMILY and TYPE model names used

Algorithm **Family**

LEVEL — **Definitions of family and type names**

Algorithm	Family	Definition
OLS	OLS	Ordinary Least-Squares
L-1.4	BC-GAUHESEQ	Box-Cox Generalized AUtoregressive HEteroskedastic Single EQquation
L-2.0	BC-DAUHESEQ	Box-Cox Directed AUtoregressive HEteroskedastic Single EQuation
		(HG : with Heteroskedasticity of a General form)

SHARE /PROBABILITY — **Definitions of family names**

Algorithm	Family	Definition
S-1/P-2	LOGIT	LOGIT
S-2/P-3	S-DOGIT	Standard DOGIT
S-3/P-4	G-DOGIT	Generalized DOGIT
S-4/P-5	LIN-IPT- LOGIT	LINear INVERSE POWER TRANSFORMATION-LOGIT
S-5/P-6	BT-IPT- LOGIT	Box-Tukey INVERSE POWER TRANSFORMATION-LOGIT

Definitions of type names for all SHARE and PROBABILITY families

S-LIN : with Standard LINear utility functions
G-LIN : with Generalized LINear utility functions
S-BC : with Standard Box-Cox utility functions
G-BC : with Generalized Box-Cox utility functions

Pre-existing first versions of the L-2.0 Family of the LEVEL class and the P-3 to P-6 Families of the PROBABILITY class were further developed and finished as part of the STEMM project (1996-1998).

2.2. Program development of the LEVEL algorithm

A. Background and baseline

The fixed part. We start from (10), restated here for convenience, and develop a model of residuals :

$$y_t^{(\lambda_y)} = \sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + u_t \quad . \tag{17}$$

Any model of residuals can be seen both as a way of extracting systematic information from residuals and as a way of fulfilling the statistical requirements of constant variance (homoskedasticity) and independence of errors needed for adequate estimation of standard errors of parameters. In this respect the literature is unbalanced because there is little emphasis on heteroskedasticity and much emphasis on autocorrelation, particularly of the serial kind in time-series. The reason is of course that extracting information from residuals is a modelling exercise that requires intuition about mispecification of the fixed part that would be expected to produce systematic patterns within the errors u_t.

The stochastic part : heteroskedasticity. Take first the statistical requirement that the u_t should have a constant variance σ^2. It is obvious that maximisation of the likelihood of the observations—the heart of which consists of minimisation of the residual sum of squared errors—cannot be done under the assumption of constancy of the variance of these errors because changing λ_y in (17) directly affects this error variance : indeed, that is precisely the reason why log-log models are often used as they yield less variable residuals, residuals with less « σκεδασισ » or dispersion !

Concerned by this problem, Zarembka (1974) developed an approximate diagnostic statistic. But more was needed for a satisfactory solution : as one cannot modify the functional form without modifying this « skedasis », the only way out is to have a separate tool to establish constant variance simultaneously with the establishment of the proper BC form of the fixed part—one cannot expect to « control » for functional form and heteroskedasticity without as many tools as there are objectives. The simplest way to think about the problem is to look for a function of a vector of variables Z_t, in the following transformation from the heteroskedastic error u_t. to the homoskedastic error v_t. :

$$u_t = \left[f(Z_t) \right]^{1/2} \bullet v_t \tag{18-A}$$

and it is practical to write a very general function for $f(Z_t)$, such as the BCG form in Gaudry and Dagenais (1978) that includes this useful special case that not only guarantees positive variances but also includes many classical forms of heteroskedasticity as special nested cases :

$$u_t = \left[\exp\left(\sum_m \delta_m Z_{mt}^{(\lambda_{z_m})} \right) \right]^{1/2} \bullet v_t \, . \tag{18-B}$$

Now it has to be kept in mind that this form (18-B) is extremely powerful also in the numerical sense that it can modify the correlation among the transformed variables of the homoskedastic expression obtained by replacing u_t in (17) by its value in (18-B), that is by effectively dividing all observations of the dependent and all right-hand side variables of (17) by $[f(Z_t)]^{½}$. Alternatively, one could say that the transformation to homoskedasticity can create « outliers » that could dominate the estimation. In that sense, and in view of the difficulty of writing a model with meaningful variables in (19), it is often found in practice and stated, for instance in Dagenais *et al*. (1987), that correcting for heteroskedasticity does not help, a result also noted by Mishkin (1990). Dagenais *et al*. also note that using a BC may create heteroskedasticity and that it is consequently false to assume that use of the BC automatically induces homoskedasticity and normalises the error distribution.

<u>*The stochastic part : serial autocorrelation*</u>. In the L-1.4 algorithm (Liem *et al*., 1987-1993), a multiple-order autocorrelation scheme is added to correct for serial autocorrelation of the v_t :

$$v_t = \sum_l \rho_l v_{t-l} + w_t \tag{19}$$

This vector of autoregressive ρ_l parameters is used even for residual errors that are clearly autoregressive moving average processes (ARMA), as indicated by the Box-Jenkins analysis provided. The reason for increasing the order of the AR scheme to model truly ARMA residuals is computational convenience. In addition, note that a multiple order generalisation of the single order AR case amounts to imposing on all variables of the model the same AR structure. This may seem restrictive to those who wish to apply a distinct AR structure to each variable in order to isolate the white noise residuals or « innovations », and then apply regression to these residuals in a second distinct step. However, our procedure has the advantage of estimating the regression coefficients and the autoregressive structure simultaneously, a process that may be more robust if many of the variables contain errors of observation (Dagenais, 1994) : clearly, whitewashing data series that contain observation errors runs the risk that the residuals of each variable will primarily consist of errors of observation. One would then expect regressions made on such « innovations » to be very sensitive to minute changes in specifications of the first step one-by-one whitewashing schemes (in practice not always themselves unique or straightforward) and to the second step choice of « regressors ». The L-1.4 algorithm simultaneously estimates the system (17)-(18) and (19), ignoring the first *l* observations to avoid creating outliers.

<u>*The stochastic part : directed autocorrelation, in particular spatial*</u>. The idea of the L-2.0 algorithm is to generalise (19) by allowing any particular error v_t to be correlated not just with a particular value of the same vector situated at a constant time distance, but with many values of v_n , for each of 2 orders:

$$v_t = \sum_{l=1}^{2} \rho_l \left(\sum_{n=1}^{N} \tilde{r}_{l,tn} v_n \right) + w_t \tag{20}$$

where $\tilde{r}_{l,tn}$ is the typical element of the matrix $\tilde{R}_l$, as may be clearer in matrix form :

$$v = \sum_{l=1}^{2} \rho_l \tilde{R}_l v + w . \tag{21}$$

Residue Impact Criteria. This matrix $\tilde{R}_l$ results from three steps. In the first step, a matrix R_l is defined with typical element $r_{l,tn}$ equal to one if there is a correlation between two residuals v_t and v_n and equal to zero otherwise. This matrix R_l therefore embodies an assumption about the nature or source of the potential correlation the presence of which will be tested by the coefficient ρ_l. The formulation of such hypotheses forms part and parcel of the model and gives rise to a terminology. Gaudry and Blum (1988) call a particular equidistance rule or assumption about the interaction among residuals a « residue impact criterion » (RIC) and distinguish between criteria arising from the natural ordering (NO) of the data—involving say time or space—and criteria that involve the researcher's directed ordering (DO) of the data—according to a behavioural or other socio-economic hypothesis. As many as two NORIC, DORIC or mixed criteria can be simultaneously used in (20). The resulting matrix R_l is called a contiguity matrix because the spatial case is the oldest. Is has long been the practice (Ord,

1975) to row or column normalise this matrix : Bolduc (1987) has shown that the new normalised matrix $\overline{R}_l$ garantees a convex likelihood function over the stable unit interval of ρ_l. Whence a second step. The formulation of RIC is « idiot-proof » in the sense that, as it involves only zeroes and ones, it avoids the great complications, such as those linked to units of measurement, that arise when elements of the matrix R_l are functions of variables, for instance distance (Bolduc *et al*., 1989). However, something is needed to compensate for the discreteness of the Boolean matrix R_l, as many phenomena are likely to be smooth and their representation by an « all-or-nothing » slice insufficient : the solution resides in taking due account of the whole set of near and distant neighbours.

Near and distant neighbours : degrees of contiguity. Following the Blum *et al*. (1990 ; 1995/1996) approach, powers of $\overline{R}_l$ generate a sequence of contiguity matrices ($\overline{R}_l^2, \ldots, \overline{R}_l^c, \ldots$) which define degrees of neighbourliness or proximity ($\overline{R}_l^2$ denoting neighbours of neighbours, and $\overline{R}_l^c$ higher powers). It is convenient to assume that the impact of these close and distant neighbours decreases geometrically with « distance », as in Koyck distributed lags of time-series (whence the name Autoregressive Contiguous Distributed (AR-C-D) for this analogue process), which leads to :

$$\tilde{R}_l = \pi_l \left[I - (1 - \pi_l)\overline{R}_l \right]^{-1} \overline{R}_l, \qquad (0 \prec \pi_l \leq 1) \ , \tag{22}$$

where the proximity parameter π_l allows endogenisation of the relative importance of near and distant effects. If π_l equals one, $\tilde{R}_l$ is equal to $\overline{R}_l$, indicating that only the adjacent neighbours have an impact on the correlations among the associated residuals assumed in defining the RIC : this corresponds exactly to the classical Ord (1975) case. By contrast, as π_l tends towards zero, the near effect is reduced to a minimum in favour of the distant effect. Generally, the π_l weigh the *relative* importance of near and distant effects with a single parameter defining the sharpness or slope of the decline.

Spatial complements and substitutes. In effect, the Boolean R_l matrix captures the impact of mispecification of the fixed part of the model, and primarily the effect of missing variables. For instance, in the Blum and Gaudry paper (1990) on the savings behaviour of German households mentioned above, the BC corrects signs as various DORIC simultaneously indicate how socio-economic groups imitate or avoid « doing like the Joneses ». In a first series of single-country spatial applications to Canada and Germany, Gaudry *et al*. (1994) show the relevance of the approach using both NORIC and DORIC specifications considered separately and jointly. They also point out that a positive ρ_l can be interpreted as implying spatial competition (substitution) among origin-destination pairs and that a negative ρ_l can be interpreted as an indication of spatial complementarity. By contrast, Wills' (1986) Generalized Gravity-Opportunities model, used recently by Leore (1996a, 1996b) also introduces interdependance among flows, but excludes detection of complementarity among them.

B. Recent developments of the L-2.0 algorithm under STEMM (1996-1998)

Although we had a first version of the L-2.0 algorithm, it was limited due to the computational burden of maximising the likelihood function for the system (17)-(18) and (20). This burden arises from the fact that the function to be maximised, as documented in Liem *et al*. (1998), contains the logarithm of the Jacobian of the transformation from w to v (from (21)), namely

$$ln \left| det \left[I - \sum_{l=1}^{2} \rho_l \tilde{R}_l \right] \right| \tag{23}$$

which means that repeated inversion of $\tilde{R}_l$ cannot be avoided throughout the already very non-linear maximisation. The Canada and Germany tests mentioned above involved some 300 origin-destination flows. Much work was therefore required to improve the speed of the algorithm and its capacity to handle large numbers of observations (up to 2000). Despite improvements of at least an order of magnitude, some tests with about 600 origin-destination flows (implying $\tilde{R}_l$ matrices of dimension 600 by 600), reported on as a STEMM « MAP-1 Additional Non-Linearity and Spatial Competition Demonstration » below, took more than 13 hours on recent SUN Ultra-60 stations.

C. Fridström's count data use of LEVEL : Box-Cox Poisson regression

Although we have here no formal discussion of Count data and have defined no class for them in Table 4, it is relevant to note that the LEVEL algorithms can be used to estimate Poisson models with BC transformations on the explanatory variables, in the spirit of PF-1 stated at the end of Section 1.1. Fridström (1997) calls his procedure to do this the Iterative Reweighted POisson-SKedastic Maximum Likelihood (IRPOSKML) method.

It consist in adding a small BT constant a to the logarithm of the dependent variable in (19) :

$$ln(y_t + a) = \sum_k \beta_k X_{k,t}^{(\lambda_{X_k})} + u_t \qquad (24)$$

and then, because of the assumed Poisson nature of y_t, of recognising that (24) will be heteroskedastic in a particular way, namely :

$$var(u_t) = var\left[ln(y_t + a)\right] \qquad (25)$$

so that, provided one can numerically evaluate the variance of the log of a Poisson variable with a small (BT) constant added as a function of $E\left[y_t\right]$—an exercise he supplies in his Appendix A—, one can iterate as follows :

(i) obtain an estimate of the dependent variable in (24) under homoskedasticity assumptions ;
(ii) find the implied $\hat{y}_t$ by « unrolling » the transformation ;
(iii) transform it by numerical approximation into $\hat{\sigma}_{[ln(y_t+a)]}$, the estimate of the right-hand side of (25) ;
(iv) use this estimate as a Z_1 variable in (20), setting $\lambda_{Z_1} = 0$, $\delta_{Z_1} = 1$ *and* $Z_1 = \hat{\sigma}_{[ln(y_t+a)]}$;
(v) find a new set of $\hat{y}_t$ and iterate until convergence (3-4 times appear sufficient).

This non-trivial procedure replaces the zeroes of count data sets by a small value and cycles on (24) until the Poisson assumption holds, taking due account of the relationship (25). Although the variance of the dependent variable and the variance of the error term are identical in a linear model, they are distinct in (17) where $\sigma^2_{y_t} \neq \sigma^2_{u_t}$, but IRPOSKML is a correct procedure because $\sigma^2_{[ln(y_t+a)]} = \sigma^2_{u_t}$.

2.3. Modal split algorithms

A. Background and baseline

The root model, the fixed part and the data class. We start from the Logit (Standard Linear) root :

$$P_i = \frac{exp(V_i)}{\sum_m exp(V_m)}, \qquad i, m = 1, \ldots, M \tag{26}$$

$$V_i = \beta_{i0} + \sum_n \beta_{in}^i X_n^i + \sum_s \beta_{is} X_s + u_i \tag{27}$$

where we have changed the notation to identify the X_n^i (where the upper index denotes the mode and the lower ones the utility function index and the network variable), the network characteristics that belong to a particular mode and therefore vary across alternatives, and it is clear that in (27) the X_s denote socio-economic characteristics of consumers that are common across alternatives.

This model is referred to in Table 4 as the Standard Linear type because it is linear in the X_k variables and these variables do not include the network characteristics of other alternatives than the i^{th} : we reserve the word Generalized for representative utility functions that would contain such variables, and therefore make the model inconsistent with the IIA axiom of choice theory, for instance :

$$V_i = \beta_{i0} + \sum_n \beta_{in}^i X_n^i + \sum_n \beta_{in}^j X_n^j + \sum_s \beta_{is} X_s \tag{28}$$

whence the Standard Linear (S-LIN) and Generalized Linear (G-LIN) types found in Table 4.

The root model. But, in the root Logit (Standard Linear), the coefficients of all variables that are common across alternatives, namely the constants and the socio-economic variables, are underidentified and one can only identify differences between the structural coefficients of an alternative and those of a reference alternative, for instance alternative r :

$$\left.\begin{aligned} V_i &= (\beta_{i0} - \beta_{r0}) + \sum_n \beta_{in}^i X_n^i + \sum_s (\beta_{is} - \beta_{rs}) X_s \\ V_i &= \beta_{i0}^{\nabla} + \sum_n \beta_{in}^i X_n^i + \sum_s \beta_{is}^{\nabla} X_s \end{aligned}\right\} \tag{29}$$

which means that *t*-statistics associated with those differences tell us nothing about whether the variables have a place in the model but only about the difference between their two coefficients.

The stochastic part. Although there is an error term associated with each representative utility function, little attention has been given to the problems of heteroskedasticity—exceptions with discrete data are Domencich and McFadden (1975) and Schnetzler (1996, 1998), who convincingly argues that heteroskedasticity is generally secondary unless socio-economic factors matter, a result compatible with some distance related heteroskedasticity found for long-distance markets for Sweden (Algers and Gaudry, 1994)— and less attention still to the problems of autocorrelation (serial or directed or spatial). We shall continue this policy of benign neglect and focus on the fixed part of the model.

Data classes. Our presentation will not differentiate much between the SHARE class (when the observed dependent variables are shares) and the PROBABILITY class (when the observations are mutually exclusive categorical choices), as these have similar properties and the arborescent enrichments of the root model that we present pertain to both classes. In fact, the 5 families found in the SHARE algorithm we developed before those found in the PROBABILITY algorithm.

Five Families and their Linear and Box-Cox model types. We discuss each of the five families first under the hypothesis that the representative utility functions are of the Generalized Linear type (28) and then under the hypothesis that they are of Generalized Box-Cox (G-BC) type (Gaudry, 1978), namely

$$V_i = \beta_{i0} + \sum_n \beta^i_{in} X_n^{i(\lambda^i_{in})} + \sum_n \beta^j_{in} X_n^{j(\lambda^j_{in})} + \sum_s \beta_{is} X_s^{(\lambda_{is})} \tag{30}$$

where the term « Generalized » applies only if network variables that belong to the j^{th} alternative are used in the i^{th} function, in contrast with the the Standard Box-Cox (S-BC) type also found in Table 4 :

$$V_i = \beta_{i0} + \sum_n \beta^i_{in} X_n^{i(\lambda^i_{in})} + \sum_s \beta_{is} X_s^{(\lambda_{is})} \tag{31}$$

Linear and Box-Cox model types defined by the utility function specification. It is then possible to identify simply the new dimension brought about by the generalisation. Generalized types (either Linear or Box-Cox) at once re-establish the interdependance of utility, make the model inconsistent with the Independence from Irrelevant Alternatives (IIA) property and allow the theoretical possibility of complementarity among alternatives. This contrasts with the well-known properties of the root model of Standard Linear type : additive separability of utility, consistency with the IIA axiom—with attending equality of cross-elasticities of demand—and necessary substitutability among alternatives. In effect, Generalized types reintroduce the classical world of microeconomics where all prices appear in the demand function for each good : a system of demand equations where only own effects are allowed and matrices of price effects are reduced to their main diagonal, thereby ruling out « specific substitution effects, inferior goods and the possibility of complementary goods [...] may be plausible when goods are defined broadly, but not with a fine classification of consumer expenditures » (Goldberger, 1967, p. 31). Equations (28) and (30), which rule out none of these three reasonable properties, should be the maintained hypotheses of models : the IIA property is not generally credible and is not likely to hold in choice models (Samuelson, 1985).

Enrichment dimensions : utility, thick tails, form and coefficient identification. Table 5 presents the summary specification of the various families, using the attractiveness functions U_i and their conditions. It also summarises the new dimensions of enrichment brought about by each family (as compared to those of the Logit), showing them first under the assumption of linearity of the representative utility functions and then under the assumption of nonlinearity.

But our interest is not only in the separability but also in the presence of asymptotes of the response function that differ from zero and one, as in Figures 6-8, as well as in asymmetry of the response curve itself. We are also interested in solving the underidentification problem shown in going from (28) to (29). As we shall see, none of these very restrictive properties of the root model need be accepted, as more credible specifications exist that may contain the root specification as a nested special case to test improvements against. Our discussion will not be formal : for this the reader should consult Liem *et al*. (1997) for the SHARE algorithm baseline and Liem and Gaudry (1998a) for the PROBABILITY algorithm developments under STEMM based on the Liem and Gaudry (1993) baseline.

Table 5. Five families and their enrichment of the root Logit (Standard Linear) model

FAMILY	U_i function	Conditions
Classical Logit (Warner, 1962 ; Cox, 1970 ; Rassam *et al*., 1970, 1971):		
LOGIT	$exp(V_i)$	
Standard-Dogit (Gaudry and Dagenais, 1977, 1979):		
S-DOGIT	$exp(V_i) + \theta_i \sum_j exp(V_j)$	$\theta_i \geq 0$
Generalized-Dogit (Gaudry and Dagenais, 1978):		
G-DOGIT	$exp(V_i) + \theta_{ij} \sum_{j \neq i} exp(V_j)$	$\theta_{ii} = 0 , \theta_{ij} \geq 0$
Linear-Inverse Power Transformation-Logit (Gaudry, 1978, 1981):		
LIN-IPT-LOGIT	$\left[\varphi_i \, exp(V_i) + 1\right]^{1/\varphi_i} - \mu_i$	$\varphi_i \geq 0 \; , \; \mu_i \leq 1$
Box-Tukey-Inverse Power Transformation-Logit (Gaudry, 1989 ; Liem and Gaudry, 1993, 1997):		
BT-IPT-LOGIT	$exp\left\{\dfrac{\left[exp(V_i) + \tilde{\mu}_i\right]^{\tilde{\varphi}_i} - 1}{\tilde{\varphi}_i}\right\}$	$\tilde{\mu}_i \geq 0$

where the utility function is of Generalized Linear *type* or of Generalized Box-Cox *type* (Gaudry, 1978) :

G-LIN :	**G-BC** :
$V_i = \beta_{i0} + \sum_n \beta_{in}^i X_n^i + \sum_n \beta_{in}^j X_n^j + \sum_s \beta_{is} X_s$	$V_i = \beta_{i0} + \sum_n \beta_{in}^i X_n^{i(\lambda_{in}^i)} + \sum_n \beta_{in}^j X_n^{j(\lambda_{in}^j)} + \sum_s \beta_{is} X_s^{(\lambda_{is})}$

ENRICHMENT DIMENSIONS							
Dimension :	**Utility**	**Ignorance/captivity**	**Form**	**Identified β_k coefficients**			
IN ROOT MODEL WITH LINEAR V_i	Separable	Tails at 0 or 1	sym.	β_{i0}^{∇}	β_{in}^i		β_{is}^{∇}
DESIRED ENRICHMENT	Not separable	Thick tails	asym.	β_{i0}		β_{in}^j	β_{is}
OBTAINED ENRICHMENT IF REPRESENTATIVE UTILITY FUNCTION V_i IS OF GENERALIZED LINEAR TYPE							
LOGIT							
S-DOGIT		due to θ_i					
G-DOGIT	due to the θ_{ij}						
LIN-IPT-LOGIT	due to BTG on $exp(V_i)$	due to $(1 - \mu_i)$	asym.	β_{i0}		β_{in}^j	β_{is}
BT-IPT-LOGIT	due to BT on $exp(V_i)$	due to $\tilde{\mu}_i$	asym.	β_{i0}		β_{in}^j	β_{is}
ADDITIONAL ENRICHMENT IF REPRESENTATIVE UTILITY FUNCTION V_i IS OF GENERALIZED BOX-COX TYPE							
LOGIT			asym.			β_{in}^j	β_{is}
S-DOGIT			asym.			β_{in}^j	β_{is}
G-DOGIT			asym.			β_{in}^j	β_{is}
LIN-IPT-LOGIT							
BT-IPT-LOGIT							

Separability of utility and coefficient identification. Under a linear form of V_i, , the representative utility function of an alternative, the classical **LOGIT** model assumes that only *own* characteristics matter. The **GENERALIZED DOGIT** corrects this by introducing a measure of similarity/dissimilarity between the alternative considered and other alternatives, as is clear when the G-DOGIT is rewritten according to Restle's (1961) format :

$$U_i \equiv R_i \; exp(V_i) = exp(V_i + ln(R_i)) \tag{32}$$

by replacing R_i by

$$R_i = 1 + \sum_{j \neq i} \theta_{ij} \; exp(V_j - V_i). \tag{33}$$

There are many other models that introduce some measure of similarity/dissimilarity : among the most recent ones are Cascetta *et al.* (1996) who introduce an average measure and Fowkes and Toner (1998) who, for the STEMM freight model, specify a « commonality factor » of the form :

$$R_i = \left\{ \sum_j \left[\frac{f_{ij}(X_{in}^i, X_{jn}^j)}{\left[f_i(X_{in}^i)\right]^{0,5} \left[f_j(X_{jn}^j)\right]^{0,5}} \right] \right\}^{\beta} \tag{34}$$

In effect all these forms reintroduce the influence of near substitute alternatives in establishing the utility of the own alternative.

Because both IPT forms « wrap » the utility function V_i or the attractiveness function U_i in a non-linear envelope, the *cross* network terms X_n^j and the socio-economic terms X_s present in all V_i functions do not « cancel out » any more as common factors—their β_{in}^j and β_{is} coefficients can be identified even if the V_i function is linear : all coefficients of the Generalized Linear function are identified, including those of the constants β_{i0}.

Identification of all alternative-specific constants in (28) is in principle as important as it is difficult : many network problems of path choice, for instance for air networks, effectively require that the equality of constants be tested under various specifications of the path choice problem and the results used to forecast constants for new paths or services. These tasks can only be performed under **IPT** families indifferently using Generalized Linear or Box-Cox types and their special Standard cases. In his application to the choice of itineraries in air networks, Laferrière (1988, 1999) used itinerary-specific constants associated to markets with 2 itineraries and to markets with more than 2 itineraries, setting the φ_i of the **LIN-IPT-LOGIT** at values very close to those of the **LOGIT**.

Under a Generalized Box-Cox form of V_i, , the representative utility function of an alternative, the **LOGIT, S-DOGIT AND G-DOGIT** families make it possible to identify the coefficients of cross network terms X_n^j and socio-economic terms X_s, the β_{in}^j and β_{is} coefficients, but do not allow for the identification of all modal constants β_{i0}. As the addition of variables in the V_i functions will profoundly modify the correlation among the error terms u_i in (27), because mispecification due to missing variables is always the principal cause of error correlation across equations, one expects the **LOGIT (G-BC)** to be a substitute for the nested Logit, as implied in a recent Canadian High Speed Rail study (Gaudry and Le Leyzour, 1994) where the addition of cross-terms considerably improved the results.

Captivity to alternatives and modeler's ignorance. There are situations where it is very difficult to properly explain the tails of response functions. The **Standard DOGIT** model formulated by Gaudry and Dagenais (October, 1977) corrects this by allowing for variable asymptotes illustrated in Figure 6 (not to be confused with big regression constants that make the response « sticky » but do not affect the asymptotic behaviour). They first interpreted their new θ_i as producing analogues of income effects.

A few weeks later, in November, upon reception of this technical paper, various people pointed out various other interpretations, including the analogy to Stone's expenditure system[3] and the role of the θ_i as captivity measures (Ben-Akiva, 1977, aware of McFadden, 1976). McFadden also noted in conversation[4] that he had written a similar model in 1976 but neglected to develop it because unimodality of the Likelihood function appeared difficult to prove upon an examination of the first-order conditions. In practice, this did not turn out to be a problem and plots of likelihood functions soon showed well behaved forms in urban and intercity mode choice and transit mode-of-payment models, either cross-sectional or time series (Gaudry and Wills, 1979 ; Gaudry, 1980). The simplest way (McFadden, 1981) to understand this model is to consider that one is drawing from a mixed distribution with two populations : one captive to any alternative in proportion C_i and the remaining non-captive and exercising a choice according to the Logit model, as in :

$$P_i = \frac{\theta_i}{(1 + \Sigma_j \theta_j)} + \frac{1}{(1 + \Sigma_j \theta_j)} \bullet \frac{exp(V_i)}{\Sigma_j exp(V_j)} = \frac{exp(V_i) + \theta_i \Sigma_j exp(V_j)}{(1 + \Sigma_j \theta_j)\, \Sigma_j exp(V_j)} \qquad (35)$$

$$P_i = C_i + (1 - \Sigma_j C_j) \bullet [\ \circ\ ]$$

It is the case that various people have effectively tried to limit the Logit response curve and prevent it from reaching its zero or one asymptote, thereby « compressing » it by statistical methods (Westley, 1979) as shown by Small (1981), or manually in the pre-assignment of captive riders to modes (Gunn and Bates, 1980), all in effect writing special cases of the **S-DOGIT.** More recently, Bordley (1989, 1990) has shown that the Dogit can be derived as a brand loyalty model even if buyers are not perfectly captive to one choice and Williams and Ortuzar (1982), Fry (1988) and Fry and Harris (1995) have studied its statistical properties.

An alternative interpretation is that of <u>modeler's ignorance</u>. If the absence of information means that the $V_i = 0$, then the asymptotes can be viewed as the « *a priori* ignorance » of the modeler. This perspective is applicable not only to the **S-DOGIT**, but also to the **IPT** families where the « thick tails » are best seen by putting the ϕ_i equal to 1 in the **LIN-IPT-LOGIT** model and the $\tilde{\phi}_i$ equal to 0 in the **BT-IPT-LOGIT** models, yielding respectively the captivity or ignorance levels $(1 - \mu_i)$ and $\tilde{\mu}_i$. In this perspective, the V_i introduce an « explanation » or a « meaning » and gradually reduce the *a priori* ignorance levels in all M directions (one per alternative).

For instance, Montmarquette and Mahseredjian (1985) have used the **LIN-IPT-LOGIT** available within the SHARE algorithm to explain the marks obtained by schoolchildren, an ideal non-transportation application : the schooling selection process makes it very difficult for students both to fail badly and to succeed extremely well, thereby defining « background » levels that are not properly explained within the model but are determined by it as unexplained asymptotes like those shown in Figure 7 or 8—at levels determined by the nature of the problem.

Asymmetry of the response curve. A first important distinction in the discussion of form pertains to the object of the application of transformations : they can be applied to variables or to functions.

[3] Pointed out by Daniel Leblanc, *Ecole Polytechnique de l'Université de Montréal.*
[4] Conversation in Marc Gaudry's office, before Daniel McFadden's November seminar at *Université de Montréal*.

In the above discussion of the BC and the definition of model types for all families, this transformation was applied to *variables*. In particular, we have pointed out how the BC transformation applied to variables of the **LOGIT** model made the response curve in Figure 5—shown with a variable X_k , not a V_i function, on the X-axis— asymmetric as one departed from the linear form. This approach, first implemented for the first time to allow both Generalized and Standard Box-Cox model types to be estimated within the P-2 algorithm (Liem and Gaudry, 1987, 1993) is also useful outside of transportation : Montmarquette and Blais (1987) use the Logit (S-BC) approach and the P-2 algorithm to measure attitudes towards risk and show that risk-neutrality (λ=1) cannot be rejected in their data. It sometimes also happens in transport mode choice analysis that the linear form cannot be rejected (McCarthy, 1982), but this is extremely rare.

Naturally, there have been attempts at using other transformations : Lerman and Louvière (1978) use a special case of BCG on a particular variable of a model ; similarly, R.A.T.P. and Cambridge Systematics, in the aftermath of their mode choice work (1982), developed a destination choice model that would only converge if the activity attractors used in the discrete choice model were transformed by the same special case of the BCG (double exponential, arising from $\varphi_{X_k} = 0$ on this variable).

But application of transformations to *functions* is a different matter. The **LIN-IPT-LOGIT** family is defined by applying a BTG to the attraction functions $U_i = exp\ (V_i)$ and was conceived, as stated in Section 1.1.C, from the realization that one could not directly transform the dependent variable of a Logit model and easily maintain the requirement that the choice probabilities sum to one, but that one could achieve this result indirectly by using inverse transformations on the right-hand side U_i functions of this model. The **BT-IPT-LOGIT** was written later applying the BT to the V_i functions instead to overcome a certain lack of flexibility of its predecessor : the LIN-IPT family tends to become very elongated in « the middle » (almost linear, as in Figure 7) when the tails are quite different from zero.

In both cases, asymmetry arises even if the Standard Linear model type is specified, as indicated in Table 5. The reaction curve of a mode will be asymmetric if one departs from the Logit case for at least one mode and can be specific to each mode, as shown in Laferrière and Gaudry (1993) with both Generalized and Standard Linear LIN-IPT specifications. In both Figures 7 and 8, the V_i functions are shown on the X-axis to make clear the fact that in this case the asymmetry is distinct from (and perhaps additional to) that occurring with a Standard or Generalized Box-Cox type of representative utility function.

B. Recent developments of the P-2 to P-6 algorithms under STEMM (1996-1998)

Making the 5 families available within PROBABILITY, as in SHARE. We had in SHARE a program for aggregate data available for the 5 families (S-1 to S-5), but in PROBABILITY a program for discrete data available only for the LOGIT family (P-2). Our first task was to program the 4 other families (P-3 to P-6) within PROBABILITY. A more difficult task however was to study the invariance properties of the IPT options within PROBABILITY, thus providing an indirect benefit to SHARE because the invariance questions are the same, whether one deals with aggregate or with disaggregate data[5].
Invariance of parameters in the presence of transformations. In regression analysis, one expects certain parameters to depend on units and others to be invariant to changes in units : changing a money term from francs to centimes should affect its coefficient but leave the form of the relationship unchanged. In

[5] The only differences occur within SHARE due to the fact that this program contains an option to use samples where all alternatives are not available. This option requires defining restrictions on the square variance-covariance matrix Σ of order M-1 of residual errors obtained by the iterative Berkson-Theil approach used, as explained in Liem *et al.* (1993, 1997).

logit models, coefficients of categorical variables (dummy variables) are invariant under arbitrary scale shifts only if alternative specific constants are present (Tardiff, 1978).

The regression β_k and the BC form. In least-squares regression, the regression β_k adjust proportionately to changes in units of the X_k. Similarly, it has long been known (Schlesselman, 1971) that the BC is invariant to a scalar transformation of a strictly positive variable as long as there is a constant in the regression and it has been found more recently that the BC is also invariant to a power transformation of a variable (Gaudry and Laferrière, 1989) even without a constant in the regression.

In the case of a variable that contains observed zeroes, the invariance is preserved by the further creation within the list of regressors of an associated Boolean dummy variable that compensates for the fact that these observations are unaffected by the change in units, the BC shift of $-1/\lambda$ in effect « floating » for these observations. For this reason, all programs discussed here automatically generate this associated dummy variable when a BC is used on these variables, called « quasi-dummy variables » in the jargon of the program documentation for LEVEL, SHARE and PROBABILITY.

Student's *t*-statistics. However, *t*-statistics of the β_k are another matter. Ever since they were invented by Gosset, they are expected to be invariant to units of measurement because estimates required to define them both adjust in due proportions. Unfortunately and to the great surprise of many whose previous results became conditional upon units of measurement they had used, Spitzer (1984) pointed out that this invariance of the *t*-statistics of the β_k does not hold in the presence of BC. For this reason, in both models of levels and split models one has to use either likelihood ratio tests, which are exact, or use estimates of *t*-statistics of the β_k that are conditional upon the estimated values of the BC in the model. In models of levels, this also holds for the *t*-statistics of the δ_m in (18-B) but not for the other parameters of the models, the λ_y , λ_X , λ_Z and the ρ_1 : their unconditional *t*-statistics do not depend upon the units of measurement of the X_k and Z_m regressors.

Invariance of results in IPT families : should variables or functions be transformed ? However, a more complex problem of lack of invariance arises with BCG and BTG transformations used to define the IPT options because these transformations are used on complete functions within a split format. In these conditions, it is almost impossible to study analytically the invariance problems that arise when one changes the units of measurement of the X_k used in the utility functions : numerical methods have to be used. We therefore studied the invariance properties of all β_k coefficients not related to the change, and of other elements (captivity/ignorance and form parameters, choice probabilities and log-likelihood values) under the very general conditions of the G-BC model types available in the programs and allowed BC transformations to be used both on continuous and strictly positive X_k variables and also on variables that contain some observed zeroes. We also considered all cases of interest : those with M specific constants in the V_i functions, as well as those with restrictions on those M constants.

The most interesting results pertain to the case of M specific constants because this case is, as pointed out above in the discussion of identification, unique to the IPT families : all other families suffer from some underidentification of the β_{i0} and only permit differences β_{i0}^{∇} to be identified. In that case, we found as expected that results are invariant to changes in measurement units of a linear X_k variable even if all envelope parameters that define the family are allowed to differ across alternatives. However, if that variable has a BC, changing its units yields invariant results only if all BCG and BTG envelope form and ignorance/captivity parameters are constrained equal across alternatives : if they are not so constrained, but are free to differ across alternatives, changing the scale of X_k matters, unfortunately. In this case, asymmetry defined as the reaction of P_i to V_i , can only exist if it is of the same shape for all alternatives. Alternative-specific asymmetry requires linear variables. In the presence of M-1 constants, only linear forms of X_k yield invariant results. We documented all findings in full.

2.4. A short summary of additional parameters

It is interesting to ask how many new parameter dimensions are added when the root models are compared to the more general specifications. In Table 6, one notes that 6 dimensions are added within any model class, neglecting differences in the meaning of parameters across families of a class and Σ.

Table 6. Parameter dimensions added to root models

	Part of the model subjected to enrichment					
	Fixed part			Stochastic part		
CLASS	Form	Utility	Ignorance	Heteroskedasticity	Autocorrelation	Var.-Cov.
LEVEL	λ_y , λ_X			δ_Z , λ_Z	ρ_l , $\tilde{\pi}_l$	
PROBABILITY	ϕ_i , λ_X	$\beta_{i,0 \ or \ s}$, θ_{ij}	θ_i , μ_i			
SHARE	ϕ_i , λ_X	$\beta_{i,0 \ or \ s}$, θ_{ij}	θ_i , μ_i			Σ
Reference	*Equation (17) and Table 5*			*Equation (18)*	*Equations (19)-(20)*	*Footnote 5*

2.5. The QDF utility

A. Background and baseline

The notion of elasticity. The notion of elasticity used to express model results since Marshall invented it in Palermo in 1882 increases in importance when transformations of variables or of functions are used because it becomes almost impossible to make intuitive sense of parameters that are not directly translatable as marginal effects as they are in the ordinary least-squares format—providing one can keep track of the units of measurement of the X_k and y ! In fact making sense of results by measures that get rid of units becomes necessary, in complicated model such as (17) or (26) and (30), to evaluate the reasonableness of estimates of the impact on *y* of changes in any explanatory variable X_k.

Sample measure of elasticity. The simplest way to present the notion is to consider the ratio of percentage changes in the two variables at a point *t*, defined for any variable X_k in function (16) as :

$$\eta_t \ (y \, , \, X_k) = \frac{\partial \ y}{\partial \ X_k} \cdot \frac{X_k}{y} \Bigg|_{X_{kt} \ , \ y_t \ , \ X_{lt}} \qquad (36)$$

where we have dropped the subscript *m* on the dependent variable and where the vertical line simply means that the derivative is « evaluated at » $y = y_t$ and $X_k = X_{kt}$ for the variables of interest and at $X_\ell = X_{\ell t}$ for other right-hand-variables belonging to (16). This expression makes it clear that even if the function (16) of interest is linear, the value of this point measure of elasticity depends on the reference levels at that point and is therefore not constant across observations. We shall explicate this intuitive notion by drawing on the formal discussion found in Dagenais *et al*. (1987) who call the above a **sample measure** because it does not recognise that *y* is a random variable due to the presence of an error term in the model.

The dependent variable *y* as a random variable. If some recognition is given of this, it is possible to define measures in terms of moments of *y*, for instance the first two moments of *y* :

$$\eta_t \left(E(y), X_k \right) = \frac{\partial E(y)}{\partial X_k} \cdot \frac{X_k}{E(y)} \Bigg|_{X_{kt},\ E(y_t),\ X_{lt}} \tag{37}$$

$$\eta_t \left(\sigma(y), X_k \right) = \frac{\partial \sigma(y)}{\partial X_k} \cdot \frac{X_k}{\sigma(y)} \Bigg|_{X_{kt},\ \sigma(y_t),\ X_{lt}} \tag{38}$$

which can be called the **expected value** measure and the **standard error** measure of elasticity and are relatively straightforward to specify and calculate in the case of models of levels. However, if share or probabilistic models are used, the mechanical application of (36)-(38) would simply lead one to specify

$$\eta_t \left(P_m , X_k \right) = \frac{\partial P_m}{\partial X_k} \cdot \frac{X_k}{P_m} \Bigg|_{X_{kt},\ Pm_t,\ X_{lt}} \tag{39}$$

for the sample measure and analogous forms for the expected value and standard error elasticity measures. But there seems to exist no such calculation in terms of moments in the case of probabilistic or share models, due to the extreme difficulty of writing density functions for these choice models.

Sample measures of « percentage points » and « probability points » for split models. Limiting ourselves in those circumstances to sample measures for these model classes, there exists a measure that is more faithful to the original notion of elasticity, and therefore more appropriate, than (39) : that of *percentage points* for share models and of *probability points* for probability models (at a point *t*):

$$\pi_t \left(P_m , X_k \right) = \frac{\partial P_m}{\partial X_k} \cdot \frac{X_k}{1} \Bigg|_{X_{kt},\ X_{lt}} \tag{40}$$

which is clearly preferable to (39) because the dependent variable of these models is already a percentage or a probability ! Although it is still conventional to compute (39), it should make more sense to compute the change in percentage points or in probability points relative to a percent variation of X_k. In practice, researchers have to keep in mind the sample values of mode shares when they decide on the reasonableness of estimates obtained with (39) : with small market shares, values much larger than one are frequent and reasonable, but they are not for a mode that dominates the market.

Summary values types: mean of the measure or measure at the mean?

How does one provide summary measures that avoid listing all points of evaluation *t = 1, ..., T* ? There are two approaches : the first and oldest one evaluates the expressions at the mean of the sample and the other calculates the (perhaps weighted) mean of point measures over the whole sample. Limiting ourselves to the sample measures and neglecting the expected value and standard error measures, for models of levels and split models, we have, respectively :

$$\eta\ (\bar{y}\ ,\ \bar{X}_k\ )\ =\ \eta_t\ (\ y\ ,\ X_k\ )\ |\ X_{kt}\ =\bar{X}_k,\ y_{t\ =}\ \bar{y}\ ,\ X_{lt}\ =\ \bar{X}_l \qquad (41)$$

$$\bar{\eta}\ (\ y\ ,\ X_k\ )\ =\ \Sigma_t \frac{\eta_t\ (\ y\ ,\ X_k\ )}{T}\ |\ X_{kt}\ ,\ y_t\ ,\ X_{lt} \qquad (42)$$

$$\eta\ (\bar{P}_m\ ,\ \bar{X}_k\ )\ =\ \eta_t\ (\ P_m\ ,\ X_k\ )\ |\ X_{kt}\ =\bar{X}_k,\ P_{mt}=\bar{P}_m\ ,\ X_{lt}\ =\bar{X}_l \qquad (43)$$

$$\bar{\eta}_w\ (\ P_m\ ,\ X_k\ )\ =\ \frac{\Sigma_t\, \hat{P}_{mt}\ \eta_t\ (\ P_m\ ,\ X_k\ )}{\Sigma_t\, \hat{P}_{mt}}\ |\ X_{kt}\ ,\ P_{mt}\ ,\ X_{lt} \qquad (44)$$

where (41) and (43) are measures of elasticity at the mean (ELM) of the sample and (42) and (44) are means of point measures of elasticity (MEL). Moreover, (44) is called a « weighted aggregate elasticity » because each point elasticity is weighted by the individual's estimated choice probability. In models of split, the expressions (43) and (44) have corresponding applications with percentage point and probability point measure (40).

Corrections for quasi-dummy and true dummy variables. As the concept of elasticity implies that an independent variable X_k is of continuous type, it needs to be modified to allow for variables that are continuous but not strictly positive—we stated above that they are called « quasi-dummy » variables in all program documentation—and for true dummy variables.

Dagenais *et al*. (1987) show that an appropriate correction in these cases consists in using mean values obtained only over positive observations of the X_k variable in question for elasticities evaluated at the mean, namely according to (41) or (43), and in using in the computation of the sum only observations (individuals in probability applications) for which X_k is positive for mean elasticity measures (42) and (44). Both corrections increase the resulting elasticity measure by about the inverse of the proportion of positive observations of X_k in the sample.

As a result, an elasticity corrected in this way for a quasi-dummy variable means that we are interested in the response of *y* when X_k *occurs* only. In the case of a true dummy variable, the corrected measure is a satisfactory evaluation of the discrete change in *y* due to the *presence* of the dummy variable.

As a result of these options, all programs produce results that can be read as « elasticities » for all variables including dummy variables. The user can therefore easily decide on the reasonableness of the model estimates without having to deal even with partial derivatives. But these are also computed and used notably to derive marginal rates of substitution among pairs of variables of choice models. Value of time calculations are the most common reason for these computations : with transformations, they naturally differ with trip duration, with how much time is saved and with the mode of interest.

Calculated measures. The above define a considerable number of possible measures of interest. Table 7 presents those currently available in the LEVEL, SHARE and PROBABILITY algorithms. Note that some are not defined in the presence of directed autocorrelation.

Table 7. Currently available elasticity, percentage point and probability point measures

CLASS :	LEVEL			SHARE	PROBABILITY	
	No autocorrelation/ serial autocorrelation	Directed autocorrelation				
Type→ Measure	ELM $\eta\,(\bar{y}, \bar{X}_k)$	ELM $\eta\,(\bar{y}, \bar{X}_k)$	MEL $\bar{\eta}\,(y, X_k)$	ELM $\eta\,(\bar{P}_m, \bar{X}_k)$	ELM $\eta\,(\bar{P}_m, \bar{X}_k)$	MEL $\bar{\eta}_w\,(y, X_k)$
$\eta(y)$	X	X		X	X	X
$\eta(E(y))$	X	not defined	X			
$\eta(\sigma(y))$	X	not defined				
$\pi(P_m)$				X	X	X

<u>*Combining level and split models*</u>. The QDF utility combines level and split models. As (3) is a product, η, the elasticity of demand of mode T_m with respect to any variable X_k, can be decomposed between its impact on p_m, mode share, and its impact on T, total demand irrespective of mode :

$$[\ \eta\ of\ Mode\ ] = [\ \eta\ of\ Total\ ] + [\ \eta\ of\ Share\ ], \qquad \text{or}$$

$$\eta\,(T_m, X_k) = \eta\,(T, X_k) + \eta\,(p_m, X_k), \tag{45}$$

more precisely :

$$\underbrace{\eta\,(\mathrm{T_m}, \mathrm{X_k})}_{(F)} = \underbrace{\underbrace{\vec{\eta}\,(\mathrm{T}, \mathrm{X_k})}_{(A)} + \underbrace{\eta(\mathrm{T}, \mathrm{U})}_{(B)} \bullet \underbrace{\eta(\mathrm{U}, \mathrm{X_k})}_{(C)}}_{(E)} + \underbrace{\eta\,(\mathrm{p_m}, \mathrm{X_k})}_{(D)}, \tag{46}$$

where (A) denotes the elasticity of T with respect to X_k , given a fixed U, subcomponent (B) is the elasticity of T with respect to U and (C) denotes the elasticity of U with respect to X_k.

The QDF utility allows one to combine measures from Table 7 to compute explicitly all components of (46) irrespective of the class, family and model type of the mode choice model and whether the level model is estimated with autocorrelation or not.

<u>B. Recent developments under STEMM (1996-1998)</u>

As STEMM involved extending the P-2 algorithm to other family members than the Logit, the range of possibilities in the QDF utility (Liem and Gaudry, 1998b) to compute the elements of (46) increased considerably. We also discovered a consistency requirement that had not been obvious in the baseline version of the utility : if a discrete choice model is used for the mode choice model, the MEL-type weighted aggregate elasticity can only be used in component (D), and cannot be used in (C). Component (C) should then only be computed with ELM-type elasticity $\eta\,(\bar{P}_m, \bar{X}_k)$: it cannot be computed in terms of a weighted aggregate measure since the utility index U represents the sum of the representative utility components across the modes for which the modal weights are not defined.

3. A first multicountry application for passengers

3.1. A multicountry strategy

A. Data foundations and simple specifications at a NUTS-3 level

In the absence of Europe-wide passenger travel surveys, the development of a multicountry model on the NUTS-3 level defined by EUROSTAT (1992) requires a specification that meets the lowest common data availability denominator across national databases that contain cross-country trips. Despite considerable, and sometimes vain, efforts—that still provided us with national data sets for various countries (Norway, Sweden, Germany)—we were limited in the availability of data sets that contained significant cross-border trips : this is not a recent problem (COST 305, 1988). The best such recent bases are the French « Enquête Transports 1993-1994 » (INSEE, 1993), the British Office for National Statistics (ONS) Channel Tunnel related International Passenger Surveys (IPS ; 1991 and 1996) and the Heathrow-related Civil Aviation Authority (CAA) data sets. Given more resources and time, this tripod could have been marginally expanded with other, more limited and partial sources.

For « intercity » travel demand modelling purposes, the NUTS-3 level (« *Départements* » in France, « *Kreise* » in Germany and « Counties » in The United Kingdom, etc.) constrains the availability of socio-economic variables for our two model stages. This means that the mode choice model cannot easily contain many socio-economic variables of the kind found in recent discrete choice models. Fortunately, a model based on network variables can be specified : it will resemble the early two-stage models outlined in Chapter 1. To the extent that there has been a drift over time of variables into the mode choice model from other stages of the decision process, and that this drift has not been determined on statistical grounds by procedures that would jointly determine the role of all variables in each stage, a simple specification in terms of modal cost and time characteristics should be acceptable. Similarly, the Generation-Distribution model will be restricted in its potential list of factors but, as it is notoriously difficult to do better than a population based gravity model at any time, this limitation is relatively inconsequential. Overall, such a structure should produce a robust model.

B. Baseline model, updates and the resulting MAP-1 model

In the terminology of Table 3, the baseline model for Germany (Gaudry *et al*., 1994) was a Level-Share combination. The Level-Probability combination made possible by available survey data provided an opportunity to make the best use of the relative performance of aggregate and disaggregate data. So two series of models were studied with national data, primarily for Germany (*Mobilität* 1991 data) but also with other national databases, to study the robustness of the intended format and specification. Each stream of work aimed at doing at least as well, for each of 3 trip purposes, as the baseline model had done without a distinction by trip purpose, using the TRIO program (Gaudry *et al*., 1993-1997).

Validated approaches were then applied to multicountry data sets. Selected results for model variants presented in Columns produced by the Tablex utility of TRIO include, in three sections, subsets of :

- in Part I, coefficients, partial derivatives, elasticities with corresponding *t*-statistics of the underlying β_k or δ_Z , as well as other indicators (genericity of β_k ; presence of a λ_k). Code names underlined once denote quasi-dummy variables and those underlined twice denote true dummy variables ;
- in Part II, coefficients of form or captivity (BC, BT, BCG and BTG), of heteroskedasticity and directed autocorrelation, some shown only with their usual *t*-statistic against 0, but others, like the proximity parameter π , shown also with the *t*-statistic against 1 ;
- in Part III, general statistics, especially the sample size and the log-likelihood value.

<u>*Mode choice : asymmetry of response*</u>. The first stream of work resulted in the generic mode choice models based on the notion of principal mode, shown in Table 8. Genericity means that the weights

associated to the network variables are constrained equal, a procedure that, in the general literature, is often deemed necessary to obtain reasonable signs. Although all definitions of variables in Table 8 are self-explanatory, it should be remembered that out-of-vehicle time is a component of total travel time and therefore captures the *supplementary* disutility of this kind of time. There is no dummy variable to account for heterogeneity among data sources. Note that about half of the huge sample of about 78 000 observations pertains to vacation trips.

For each trip purpose, the first Column (MAP 1.1, MAP 2.1 and MAP 3.1) of Table 8 (or see Appendix 1 for more details), shows results for the usual linear model and the second Column (MAP 1.2, MAP 2.2 and MAP 3.2) shows the impact of a single Box-Cox transformation. In the three models, this BC induces astronomical gains in log-likelihood value, more significant explanatory variables and implies an asymmetric response function, the first such demonstration for a multi-country model.

Generation-Distribution : rejection of the multiplicative homoskedastic form. The second stream of work improves the standard formulation of distribution models in three ways : (i) the population term at the origin is weighted by the propensity to travel by age class, an idea first developed in Last (1998) and applied here (with fixed weights derived from the French data) ; (ii) the impedances include both language and the presence of a border, in addition to the utility term from a Logit model (the small improvement over a simple travel time measure is documented in Appendices 2-4) ; (iii) interesting attractors are defined for vacation trips. The sample of 21 000 Origin-Destination flows contains 40 % of private purpose trips and is strongly home-based in France and the United Kingdom (Appendix 5).

Appendices 2-4 show the impact of progressively introducing Box-Cox transformations and heteroskedasticity (of the general type (18-B)—and very convincingly based on distance). Table 9 summarises the work found in these appendices by focusing on the contrast between the usual multiplicative gravity-like homoskedastic specification for each trip purpose (MAP 1.1, MAP 2.1 and MAP 3.1) and the more general BC-HE (Box-Cox heteroskedastic) formulation (MAP 1.2, MAP 2.2 and MAP 3.2). The multiplicative form, including the famous log-sum aggregator (12), is rejected in all models despite the fact that the actual values of the BC are numerically quite close to zero.

There may perhaps be an exception to this general conclusion in the vacation model (MAP 3.2) where low *t*-statistics with respect to zero appear to contradict the likelihood ratio tests results. In this case, it is possible that the approximate method (Berndt *et al*., 1974) used to compute *t*-statistics from first derivatives of the log-likelihood is too imprecise when the true value of the coefficient is close to zero—in which case the *t*-test lacks power anyway—and that one should not go beyond the exact likelihood ratio test for this model. This minor point need not detract from the overall value of this powerful example : the generalisation of the multiplicative form would be impressive in a national model and is of no less interest in this first-ever context of a multinational model.

Detailed examination of the results found in Table 9 are also interesting :

- the sums of population and GDP per person elasticities is less than 1, and differs across purposes;
- although all elasticities of the utility term increase with the BC, private trips are always and remain the most sensitive, twice as high (at 0,54) as the other trip purposes (at 0,23 and 0,24) ;
- border effects are larger for business than for other purposes and smallest for vacation trips, but language effect are inexistant for business trips and strongest for vacation trips ;
- Channel survey flows are much lower than other flows, reflecting perhaps both sampling design and geographical barrier effects.

Table 8. MAP-1 Reference Asymmetric Response Mode Choice Models (F. Heinitz, July 1998)
LOGIT Family
Standard Linear and Standard BOX-COX Types (PROBABILITY P-2 Algorithm)
Choice Probability for Three Modes; Business, Private and Vacation Trips (CAA/IPS 1996 data)

I. W.A. ELASTICITIES and (COND. T-STATISTIC)							
	VARIANT =	MAP 1.1	MAP 1.2	MAP 2.1	MAP 2.2	MAP 3.1	MAP 3.2
	FAMILY =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
	TYPE =	linlogit	bc1logit	linlogit	bc1logit	linlogit	bc1logit
	PURPOSE =	BUSINESS	BUSINESS	PRIVATE	PRIVATE	VACATION	VACATION
ALTERNATIVE 1 : AIR							
NETWORK VARIABLES							
TOTAL GROUP	TXCostA	-.07	-.09	-.05	-.09	-.09	-.10
COST [AIR]		(-18.61)	(-20.30)	(-10.01)	(-12.74)	(-23.78)	(-24.32)
	(1)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TTTimeA	-.05	-.14	-.05	-.14		
TRAVEL TIME [AIR]		(-14.25)	(-23.03)	(-9.47)	(-17.77)		
	(1)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)		
TOTAL	TOoVTimA	-.02	-.03	-.01	-.03		
OUT-OF-VEHICLE [AIR]		(-2.35)	(-6.59)	(-1.44)	(-2.98)		
	(1)	(GE)	(GE)	(GE)	(GE)		
REGRESSION CONSTANT	CONSTANT	-	-	-	-	-	-
		(34.31)	(31.05)	(35.53)	(35.38)	(-16.03)	(-41.14)
	(1)						
ALTERNATIVE 2 : RAIL							
NETWORK VARIABLES							
TOTAL GROUP	TXCostR	-.57	-1.40	-.15	-.88	-.20	-.20
COST [RAIL]		(-18.61)	(-20.30)	(-10.01)	(-12.74)	(-23.78)	(-24.32)
	(2)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TTTimeR	-1.00	-1.32	-.54	-.91	-1.55	-1.63
TRAVEL TIME [RAIL]		(-14.25)	(-23.03)	(-9.47)	(-17.77)	(-68.32)	(-70.66)
	(2)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TOoVTimR	-.09	-.21	-.04	-.09		
OUT-OF-VEHICLE [RAIL]	--------	(-2.35)	(-6.59)	(-1.44)	(-2.98)		
	(2)	(GE)	(GE)	(GE)	(GE)		
TOTAL NUMBER OF	TUmstR	-1.11	-1.06	-2.04	-1.73	-.17	-.18
TRANSFERS [RAIL]		(-14.25)	(-15.93)	(-24.68)	(-22.00)	(-5.10)	(-5.29)
	(2)	(SP)	(SP)	(SP)	(SP)	(SP)	(SP)
REGRESSION CONSTANT	CONSTANT	-	-	-	-	-	-
		(19.09)	(18.43)	(26.97)	(19.45)	(7.17)	(7.55)
	(2)						

Table 8. (continued)

I. W.A. ELASTICITIES and (COND. T-STATISTIC)							
	VARIANT =	MAP 1.1	MAP 1.2	MAP 2.1	MAP 2.2	MAP 3.1	MAP 3.2
	FAMILY =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
	TYPE =	linlogit	bc1logit	linlogit	bc1logit	linlogit	bc1logit
	PURPOSE =	BUSINESS	BUSINESS	PRIVATE	PRIVATE	HOLIDAY	HOLIDAY
ALTERNATIVE 4 : CAR							
NETWORK VARIABLES							
TOTAL COST [CAR]	TCostC	-.76	-1.25	-.22	-.81	-.22	-.22
		(-18.61)	(-20.30)	(-10.01)	(-12.74)	(-23.78)	(-24.32)
	(4)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL TRAVEL TIME [CAR]	TTTimeC	-1.12	-1.34	-.72	-.91	-1.60	-1.66
		(-14.25)	(-23.03)	(-9.47)	(-17.77)	(-68.32)	(-70.66)
	(4)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL OUT-OF-VEHICLE [CAR]	TOoVTimC --------	-0...	-0...	-0...	-0...		
		(-2.35)	(-6.59)	(-1.44)	(-2.98)		
	(4)	(GE)	(GE)	(GE)	(GE)		

II. PARAMETERS UNCOND.[T-STATISTIC=0] UNCOND.[T-STATISTIC=1]							
BOX-COX TRANSFORMATIONS							
LAMBDA(X) - GROUP 1	LAM 1	1.00	-.62	1.00	-.33	1.00	-.53
		FIXED	[-13.39]	FIXED	[-7.47]	FIXED	[10.79]
			[-34.99]		[-29.84]		[-9.51]

III.GENERAL STATISTICS							
	VARIANT =	MAP 1.1	MAP 1.2	MAP 2.1	MAP 2.2	MAP 3.1	MAP 3.2
	FAMILY =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
	VARIANT =	linlogit	bc1logit	linlogit	bc1logit	linlogit	bc1logit
	PURPOSE =	BUSINESS	BUSINESS	PRIVATE	PRIVATE	HOLIDAY	HOLIDAY
LOG-LIKELIHOOD	-FINAL VALUE	-6731.15	-6246.86	-7288.48	-7117.35	-20343.0	-20299.7
	-INITIAL VALUE	-7286.77	-24563.9	-7926.74	-7926.74	-25422.2	-25422.2
	-WITH CONSTANTS ONLY	-7286.77	-7286.77	-7926.74	-7926.74	-25422.2	-25422.2
	-RATIO TEST	1111.24	2079.82	1276.52	1618.77	10158.46	10245.05
RHO-SQUARED		.08	.14	.08	.10	.20	.20
RHO-SQUARED BAR	-AKAIKE	.08	.14	.08	.10	.20	.20
	-HOROWITZ	.08	.14	.08	.10	.20	.20
	-HENSHER AND JOHNSON	.08	.14	.08	.10	.20	.20
PER CENT RIGHT		92.07	92.18	87.67	87.61	76.24	77.50
SAMPLE	-NUMBER OF ALTERNATIVES	3	3	3	3	3	3
	-NUMBER OF OBSERVATIONS	22579	22579	17477	17477	37512	37512
	-FIRST OBSERVATION	1	1	1	1	1	1
	-LAST OBSERVATION	22579	22579	17477	17477	37512	37512
	-AVAILABLE OBSERVATIONS IN VT	22359	22359	17308	17308	35004	35004
	-AVAIL OBS IN 1 AIR	22579	22579	17477	17477	37512	37512
	-AVAIL OBS IN 2 RAIL	22579	22579	17477	17477	37512	37512
	-AVAIL OBS IN 4 CAR	22579	22579	17477	17477	37512	37512
NUMBER OF ESTIMATED PARAMETERS							
	-BETAS .VARIABLES	4	4	4	4	3	3
	.CONSTANTS	2	2	2	2	2	2
	.ASSOCIATED DUMMIES	0	0	0	0	0	0
	-BOX-COX TRANSFORMATIONS	0	1	0	1	0	1

Note: under each t-statistic found in Section I, the symbols denote a generic (GE) or specific (SP) regression coefficient, as well as (L), the presence of a BCT and its group identification.

Table 9. MAP-1 Reference Flexible Form Generation-Distribution Models (J. Last, July 1998)
BC-GAUHESEQ Family (LEVEL 1.4 algorithm)
Multiplicative, BOX-COX (2-3) and Heteroskedasticity Types
Total Flow by all Modes; Business, Private and Vacation Trips (CAA/IPS 1991 and SES 1993-1994 data)

I. ELASTICITY E(Y) and (COND. T-STATISTIC)	VARIANT = / FAMILY = / TYPE = / PURPOSE =	MAP 1.1 LEVEL-1 LOG BUSINESS	MAP 1.2 LEVEL-1 BC3+HE BUSINESS	MAP 2.1 LEVEL-1 LOG PRIVATE	MAP 2.2 LEVEL-1 BC2+HE PRIVATE	MAP 3.1 LEVEL-1 LOG VACATION	MAP 3.2 LEVEL-1 BC3+HE VACATION
ORIGIN CHARACTERISTICS							
WORKING POP. WEIGHTED AT ORIGIN	wpopxpsi3 ---------	.32 (14.13) LAM 1	.33 (17.65) LAM 1				
POPULATION WEIGHTED AT ORIGIN	popxpsi3			.34 (19.35) LAM 1	.34 (21.32) LAM 1	.27 (23.05) LAM 1	.26 (24.09) LAM 1
GDP PER WORKER AT ORIGIN	o_gdppwc	.55 (8.11) LAM 3	.55 (9.43) LAM 3				
GDP PER CAPITA AT ORIGIN	o_gdppc			-.01 (-.21)	.05 (1.66)	.06 (2.32)	.10 (3.99)
AREA OF ORIGIN	o_area	-.04 (-2.06) LAM 1	-.06 (-3.23) LAM 1				
DESTINATION CHARACTERISTICS							
WORKING POP. AT DESTINATION	d_workpop	.43 (18.29) LAM 1	.44 (23.71) LAM 1				
POPULATION AT DESTINATION	d_pop			.50 (24.90) LAM 1	.50 (28.16) LAM 1	.14 (8.65) LAM 1	.13 (8.47) LAM 1
GDP PER WORKER AT DESTINATION	d_gdppwc	.50 (8.02) LAM 3	.47 (9.27) LAM 3				
AREA OF DESTINATION	d_area	-.08 (-4.15) LAM 1	-.10 (-5.69) LAM 1				
SEASIDE AT DESTINATION	seaside =======			.12 (3.52)	.13 (4.06)	.15 (5.39)	.15 (5.93)
MOUNTAINS AT DESTINATION	mountain ========					.30 (6.69)	.32 (7.02)
DESTINATION/ORIGIN RATIO							
JULY TEMPERATURE	summer2					1.07 (16.24) LAM	1.27 (17.52) LAM
IMPEDANCES							
UTILITY	util_int3	.06 (8.01) LAM	.23 (12.50) LAM	.33 (20.42) LAM	.54 (27.21) LAM	.18 (8.91) LAM	.24 (13.29) LAM
BORDER IS CROSSED	cross =====	-.82 (-7.21)	-.81 (-6.46)	-.46 (-4.13)	-.39 (-3.68)	-.33 (-5.27)	-.26 (-4.18)
COMMON LANGUAGE ACROSS BORDER	lang2 =====			.30 (1.43)	.13 (.67)	.57 (3.13)	.50 (3.38)
DATABASE							
CHANNEL SURVEY	ips_caa =======	-2.61 (-22.73)	-2.01 (-15.24)	-2.47 (-20.50)	-1.99 (-17.04)	-1.79 (-29.55)	-1.75 (-28.98)
RECORD # 1642 OUTLIER	rec1642 =======			3.09 (2.30)	3.16 (.00)		

Table 9. (continued)

		MAP 1.1	MAP 1.2	MAP 2.1	MAP 2.2	MAP 3.1	MAP 3.2
RECORD # 1912 OUTLIER	rec1912 =======					2.70 (2.86)	2.61 (.00)
ASSOCIATED DUMMIES GROUP							
WORKING POP. WEIGHTED AT ORIGIN	wpopxpsi3 =========	-3.46 (-5.57)	-2.46 (-2.49)				
REGRESSION CONSTANT	CONSTANT	- (12.21)	- (8.88)	- (2.00)	- (17.02)	- (18.97)	- (21.36)
HETEROSKEDASTICITY STRUCTURE							
DELTA COEFFICIENTS							
DISTANCE BY CAR	dist		-.17 (-10.39) LAM		.02 (4.57) LAM		.01 (10.04) LAM

II. PARAMETERS and (COND. T-STATISTIC)		VARIANT =MAP 1.1 FAMILY =LEVEL-1 TYPE =LOG PURPOSE =BUSINESS	MAP 1.2 LEVEL-1 BC3+HE BUSINESS	MAP 2.1 LEVEL-1 LOG PRIVATE	MAP 2.2 LEVEL-1 BC2+HE PRIVATE	MAP 3.1 LEVEL-1 LOG VACATION	MAP 3.2 LEVEL-1 BC3+HE VACATION
HETEROSKEDASTICITY STRUCTURE							
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]							
LAMBDA(Z)	dist		.78 [2.47] [-.68]		3.82 [2.18] [1.61]		5.56 [7.42] [6.08]
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]							
LAMBDA(Y) - GROUP 1	LAM 1	.00 FIXED	.05 [7.06] [-129.3]	.00 FIXED	.03 [5.82] [-212.1]	.00 FIXED	.01 [1.26] [-156.8]
LAMBDA(X)	util_int3	.00 FIXED	.33 [9.56] [-19.20]	.00 FIXED	.31 [14.61] [-33.35]	.00 FIXED	0... [.06] [-15.38]
LAMBDA(X)	summer2					.00 FIXED	1.02 [1.67] [.03]
LAMBDA(X) - GROUP 1	LAM 1	.00 FIXED	.05 [7.06] [-129.3]	.00 FIXED	.03 [5.82] [-212.1]	.00 FIXED	.01 [1.26] [-156.8]
LAMBDA(X) - GROUP 3	LAM 3	.00 FIXED	-.06 [-.33] [-5.41]				

III. GENERAL STATISTICS	VARIANT =MAP 1.1 FAMILY =LEVEL-1 TYPE =LOG PURPOSE =BUSINESS	MAP 1.2 LEVEL-1 BC3+HE BUSINESS	MAP 2.1 LEVEL-1 LOG PRIVATE	MAP 2.2 LEVEL-1 BC2+HE PRIVATE	MAP 3.1 LEVEL-1 LOG VACATION	MAP 3.2 LEVEL-1 BC3+HE VACATION
LOG-LIKELIHOOD	-50852.7	-50737.6	-80125.0	-79986.1	-65405.3	-65327.1
SAMPLE : - NUMBER OF OBSERVATIONS	5711	5711	8334	8334	6540	6540
- FIRST OBSERVATION	1	1	1	1	1	1
- LAST OBSERVATION	5711	5711	8334	8334	6540	6540
NUMBER OF ESTIMATED PARAMETERS :						
- FIXED PART :						
. BETAS	11	11	10	10	12	12
. BOX-COX	0	3	0	2	0	3
. ASSOCIATED DUMMIES	1	1	0	0	0	0
- AUTOCORRELATION	0	0	0	0	0	0
- HETEROSKEDASTICITY :						
. DELTAS	0	1	0	1	0	1
. BOX-COX	0	1	0	1	0	1
. ASSOCIATED DUMMIES	0	0	0	0	0	0

3.2. Tests of algorithmic developments under STEMM

Although these MAP-1 results constitute first instances of multi-country applications of the modal asymmetry and generalized gravity specifications, they are not strictly speaking tests of the most advanced program design improvements carried out under STEMM. In models of levels, these design improvements were concentrated on the addition of directed autocorrelation (notably spatial) options usable with large samples of an aggregate nature (see Section 2.2.B) ; in choice models, the design improvements featured generalizations of the Logit model to allow captivity/ignorance (thick tails) and overall asymmetry of the response curve (of the V_i representative utility function or of the U_i attraction function) with samples of a discrete nature (see Section 2.3.B).

To demonstrate the usefulness of the new designs, we performed additional tests, but only on the business trip models. To ease the comparison of results, we produced for the mode choice and generation-distribution models new tables where MAP-1 results already listed in Tables 8-9 are reproduced along with the more general results to be discussed presently.

A. Demonstration of new dimensions : mode choice

In Table 10, the first two Columns reproduce the MAP-1.1 and MAP-1.2 models of the form :

$$U_i \ = \ exp(V_i) \quad , V_i \ = \ \beta_{i0}^{\nabla} \ + \ \sum_n \beta_{in}^{i} X_n^{i(\lambda_n)} \tag{47}$$

and we note in passing that the optimal form for the X_n is $1/X^{0,6}$, almost one over square root of the network variable. The existence of a negative λ on a network variable often indicates that the *expenditure* specification of network variables [Fare, Time] might be changed to the *rate* specification [Price, Speed, Distance] in order to best capture the implicit money and time income effects that may occur in demand for long-distance trips: these specifications are only equivalent in logarithmic form.

We wish to test whether such estimates leave much to be still explained (captivity/ignorance) and properly account for the form of the response curve despite the clear presence of non-linearity on the variables. We therefore use the BT-IPT-Logit family with a representative utility function of S-BC type. We choose the BT-IPT instead of the LIN-IPT family because the latter tends to require more computing time than the former. We continue with the S-BC function from (47), without adding a third constant, to insure simple continuity with the Logit family results : the third constant would require an intermediate model and make the models non-nested because a formulation with M-1 identified alternative-specific constants is not nested in a formulation with M identified alternative-specific constants. Our demonstration model is therefore :

$$U_i \ = \ exp\left\{ \frac{\left[exp(V_i) \ + \ \tilde{\mu}_i \right]^{\tilde{\varphi}_i} \ - \ 1}{\tilde{\varphi}_i} \right\} \quad , V_i \ = \ \beta_{i0}^{\nabla} \ + \ \sum_n \beta_{in}^{i} X_n^{i(\lambda_n)} \tag{48}$$

and we propose to analyze four different variants, called D-1.3 to D-1.6 in Table 10. Due to the fact that M-1 alternative-specific constants are used (and not M), the results shown would not be invariant to eventual changes in the units of measurement of the X_n , but this is immaterial for our purposes, as the reader can infer from the great stability of the results (elasticities, BC transformations), because units are not modified as we pass from Logit Columns (where they would be invariant) to the IPT Columns.
Captivity/ignorance. First, we start from the Logit case and introduce a common $\tilde{\mu}$ on all three alternatives. The great increase of more than 13 points in log-likelihood from Column MAP 1.2 to

Column D 1.3 shows that captivity/ignorance is present. If one removes the equality constraint imposed on the individual $\tilde{\mu}_i$, further gains of 7 points are achieved (comparing Column D 1.3 and Column D 1.6). These gains suggest that the modeler's ignorance differs across the modes...

Overall asymmetry of response. Starting from Column D 1.3 where captivity/ignorance is established, the addition of a common $\tilde{\varphi}$ on all three alternatives in Column D 1.4 causes the log-likelihood to increase by 17 points : this large increase demonstrates the presence of overall response asymmetry even if the optimal value of the $\tilde{\varphi}$ — indicated in Table 10 as { 0...} but in fact more precisely stated as {0,003}— is close to zero (with a *t*-statistic of only 3.37 for the numerical reasons mentioned above). This means that, despite accounting for non-linearity on the X_n , there remains non-linearity in the response to the V_i function itself, as well as modeler's ignorance in the MAP- 1 business model

B. Demonstration of new dimensions : generation-distribution

In Table 11, the first Column D 1.1 shows estimates obtained with the same specification as that found in Column MAP 1.1 of Table 9, but with a sample reduced randomly from 5711 to 544 observations in a way that preserves the shares of the 5 data sources identified in Appendix 4. Although reduction in sample size would not be necessary to search for additional non-linearity, it is practical in a search for spatial autocorrelation of the residuals because of the need to compute repeatedly throughout estimation the inverse of matrix $\tilde{R}_l$ defined in (22), of size T by T determined by the number of observations (see Section 2.2.B). We then start again with the classical multiplicative gravity equation results of Column D 1.1 that differ slightly from those of the ancestor model MAP 1.1 (in particular, the « weighted population at origin » variable is now strictly positive, removing the need for a compensating associated dummy variable).

Generalized gravity with heteroskedasticity. In contrast with the case shown in Table 9, where three BC are used, we now use a different BC on all transformable variables (true dummy variables cannot be transformed), dependent and independent, extracting further non-linearity than if we had used only 3 BC (the test value is not shown). We then add in Column D 1.3 the generalized heteroskedasticity form with the distance variable previously used. The elasticity of the distance variable implies that the variance of the error term falls with distance, but at a decreasing rate (the BC of distance equals 0,86).

Spatial autocorrelation. In Column D 1.4, we test for spatial correlation of residuals under the NORIC rule that the typical element r_{tn} of the R matrix for a first order process (20) be equal to one if either the flow considered has the same destination and is a near neighbour of the origin, or conversely [has the same origin and is a near neighbour of the destination], and otherwise be equal to zero. This rule, applied with a definition of « near » to mean « any point in <u>a ring situated from 100 to 300 km</u> from the point considered », states that one expects errors of explanation of flows with starting or destinating points « near » (within the rings) those considered to be correlated. We did not choose circles around each point, but rings, in order to exclude any remaining short distance trips from consideration. We chose 100-300 km, instead of the 100-180 km used for Germany in the baseline model because of the multicountry nature of the application. Results of Column D 1.4 show that positive spatial correlation, indicative of competition among markets, is present, but only for close neighbours, as the proximity parameter π that defines the tail remains almost equal to 1 in Column D 1.5.

Both non-linearity and spatial correlation are then still present in the MAP-1 business model.

Table 10. MAP-1 Additional Captivity/Ignorance and Overall Response Asymmetry Demonstration
BOX-TUKEY INVERSE POWER TRANSFORMATION - LOGIT Family
Standard BOX-COX Type (PROBABILITY P-2 AND P-6 algorithms)
Choice Probability for Three Modes; Business Trips (CAA/IPS 1996 data)

I. W.A. ELASTICITIES and (COND. T-STATISTIC)							
	VARIANT =	MAP 1.1	MAP 1.2	D 1.3	D 1.4	D 1.5	D 1.6
	FAMILY =	LOGIT	LOGIT	BT-IPTL	BT-IPTL	BT-IPTL	BT-IPTL
	TYPE =	linlogit	bc1logit	bt_1L1M	bt1L1M1P	bt1L1M3P	bt_1L3M
	PURPOSE =	BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS
ALTERNATIVE 1 : AIR							
NETWORK VARIABLES							
TOTAL GROUP	TXCostA	-.07	-.09	-.09	-.09	-.03	-.08
COST [AIR]		(-18.61)	(-20.30)	(-29.56)	(-1.77)	(-2.14)	(-29.69)
	(1)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TTTimeA	-.05	-.14	-.15	-.14	-.07	-.14
TRAVEL TIME [AIR]		(-14.25)	(-23.03)	(-24.91)	(-1.76)	(-2.16)	(-23.75)
	(1)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TOoVTimA	-.01	-.03	-.05	-.06	-.49	-.09
OUT-OF-VEHICLE [AIR]		(-2.35)	(-6.59)	(-9.31)	(-1.74)	(-2.21)	(-15.34)
	(1)	(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
REGRESSION CONSTANT	CONSTANT	-	-	-	-	-	-
		(34.31)	(31.05)	(25.57)	(1.75)	(2.25)	(18.52)
	(1)						
ALTERNATIVE 2 : RAIL							
NETWORK VARIABLES							
TOTAL GROUP	TXCostR	-.57	-1.40	-1.34	-1.32	-.66	-1.47
COST [RAIL]		(-18.61)	(-20.30)	(-29.56)	(-1.77)	(-2.14)	(-29.69)
	(2)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TTTimeR	-1.00	-1.32	-1.30	-1.27	-1.07	-1.58
TRAVEL TIME [RAIL]		(-14.25)	(-23.03)	(-24.91)	(-1.76)	(-2.16)	(-23.75)
	(2)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TOoVTimR	-.09	-.21	-.30	-.32	-4.03	-.67
OUT-OF-VEHICLE [RAIL]		(-2.35)	(-6.59)	(-9.31)	(-1.74)	(-2.21)	(-15.34)
	(2)	(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
TOTAL NUMBER OF	TUmstR	-1.11	-1.06	-1.26	-1.28	-.32	-.81
TRANSFERS [RAIL]		(-14.25)	(-15.93)	(-15.40)	(-1.77)	(-2.08)	(-8.65)
	(2)	(SP)	(SP)	(SP)	(SP)	(SP)	(SP)
REGRESSION CONSTANT	CONSTANT	-	-	-	-	-	-
		(19.09)	(18.43)	(21.24)	(1.76)	(2.25)	(14.99)
	(2)						
ALTERNATIVE 4 : CAR							
NETWORK VARIABLES							
TOTAL COST [CAR]	TCostC	-.76	-1.25	-1.17	-1.22	-1.00	-1.58
		(-18.61)	(-20.30)	(-29.56)	(-1.77)	(-2.14)	(-29.69)
	(4)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TTTimeC	-1.12	-1.34	-1.27	-1.27	-1.62	-1.84
TRAVEL TIME [CAR]		(-14.25)	(-23.03)	(-24.91)	(-1.76)	(-2.16)	(-23.75)
	(4)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)	L 1(GE)
TOTAL	TOoVTimC	-0...	-0...	-0...	-0...	-.01	-.01
OUT-OF-VEHICLE [CAR]	--------	(-2.35)	(-6.59)	(-9.31)	(-1.74)	(-2.21)	(-15.34)
	(4)	(GE)	(GE)	(GE)	(GE)	(GE)	(GE)

Note: under each t-statistic of Section I, the symbols denote generic (GE) or specific (SP) regression coefficient, as well as (L), the presence of a BCT and its group identification.

Table 10 (continued)

II. PARAMETERS and UNCOND.[T-STATISTIC=0] UNCOND.[T-STATISTIC=1]							
	VARIANT =	MAP 1.1	MAP 1.2	D 1.3	D 1.4	D 1.5	D 1.6
	FAMILY =	LOGIT	LOGIT	BT-IPT	BT-IPT	BT-IPT	BT-IPT
	TYPE =	linlogit	bc1logit	bt_1L1M	bt1L1M1P	bt1L1M3P	bt_1L3M
	PURPOSE =	BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS
BOX-COX TRANSFORMATIONS							
LAMBDA(X) - GROUP 1	LAM 1	1.00	-.62	-.62	-.49	-.42	-.58
		FIXED	[-13.36]	[-19.64]	[-12.58]	[-6.28]	[-17.40]
			[-34.90]	[-51.28]	[-38.38]	[-21.30]	[-47.42]
EXTRA PARAMETERS							
PHI 1	AIR			.00	0...	0...	.00
				FIXED	[3.37]	[3.68]	FIXED
					[-1211]	[-880.8]	
					GRP 1		
PHI 2	RAIL			.00	0...	0...	.00
				FIXED	[3.37]	[3.76]	FIXED
					[-1211]	[-934.5]	
					GRP 1		
PHI 4	CAR			.00	0...	0...	.00
				FIXED	[3.37]	[3.78]	FIXED
					[-1211]	[-963.7]	
					GRP 1		
MU 1	AIR			0...	0...	0...	0...
				[.03]	[.01]	[.01]	[.03]
				[-INF]	[-INF]	[-INF]	[-INF]
				GRP 1	GRP 1	GRP 1	
MU 2	RAIL			0...	0...	0...	0...
				[.03]	[.01]	[.01]	[.03]
				[-INF]	[-INF]	[-INF]	[-INF]
				GRP 1	GRP 1	GRP 1	
MU 4	CAR			0...	0...	0...	0...
				[.03]	[.01]	[.01]	[.03]
				[-INF]	[-INF]	[-INF]	[-INF]
				GRP 1	GRP 1	GRP 1	

III. GENERAL STATISTICS							
	VARIANT =	MAP 1.1	MAP 1.2	D 1.3	D 1.4	D 1.5	D 1.6
	FAMILY =	LOGIT	LOGIT	BT-IPT	BT-IPT	BT-IPT	BT-IPT
	TYPE =	linlogit	bc1logit	bt_1L1M	bt1L1M1P	bt1L1M3P	bt_1L3M
	PURPOSE =	BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS
LOG-LIKELIHOOD	-FINAL VALUE	-6731.15	-6246.86	-6233.19	-6216.05	-5481.88	-6025.21
	-WITH CONSTANTS ONLY	-7286.77	-7286.77	-7286.77	-7286.77	-7286.77	-7286.77
RHO-SQUARED		.08	.14	.14	.15	.25	.17
RHO-SQUARED BAR	-AKAIKE	.08	.14	.14	.15	.25	.17
	-HOROWITZ	.08	.14	.14	.15	.25	.17
	-HENSHER AND JOHNSON	.08	.14	.14	.15	.25	.17
PER CENT RIGHT		92.07	92.19	92.16	92.12	92.87	91.70
SAMPLE	-NUMBER OF ALTERNATIVES	3	3	3	3	3	3
	-NUMBER OF OBSERVATIONS(ALL MODES)	22579	22579	22579	22579	22579	22579
NUMBER OF ESTIMATED PARAMETERS							
-BETAS	.VARIABLES	4	4	4	4	4	4
	.CONSTANTS	2	2	2	2	2	2
	.ASSOCIATED DUMMIES	0	0	0	0	0	0
-BOX-COX TRANSFORMATIONS		0	1	1	1	1	1
-THETA		0	0	0	0	0	0
-PHI		0	0	0	1	3	0
-MU		0	0	1	1	1	3

Note: under each t-statistic of Section I, the symbols denote generic (GE) or specific (SP) regression coefficient, as well as (L), the presence of a BCT and its group identification.

Table 11. MAP-1 Additional Non-Linearity and Spatial Competition Demonstration
BC-DAUHESEQ Family (LEVEL 2.0 algorithm)
Multiplicative, BOX-COX (8), Heteroskedasticity and Directed Autocorrelation Types
Total Flow by all Modes; Business Trips (CAA/IPS 1991 and SES 1993-1994 data)

I. ELASTICITY and (COND. T-STATISTIC)	E(y) (EP)	VARIANT = D 1.1	D 1.2	D 1.3	D 1.4	D 1.5
	FAMILY =	LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2
	TYPE =	LOG	BC8	BC8+ HE	BC8+ HE+AU	BC8+ HE+AU+PR
	PURPOSE =	BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS
ORIGIN CHARACTERISTICS						
WORKING POP. WEIGHTED AT ORIGIN	wpopxpsi3	0.42	0.31	0.30	0.32	0.32
		(5.62)	(8.39)	(10.65)	(10.05)	(9.98)
		LAM	LAM	LAM	LAM	LAM
GDP PER WORKER AT ORIGIN	o_gdppwc	0.22	0.38	0.38	0.39	0.39
		(1.03)	(1.74)	(2.60)	(2.24)	(2.24)
		LAM	LAM	LAM	LAM	LAM
AREA OF ORIGIN	o_area	0...	-0.11	-0.07	-0.06	-0.06
		(0.03)	(-1.62)	(-1.06)	(-0.93)	(-0.93)
		LAM	LAM	LAM	LAM	LAM
DESTINATION CHARACTERISTICS						
WORKING POP. AT DESTINATION	d_workpop	0.42	0.42	0.41	0.44	0.44
		(5.52)	(7.86)	(8.13)	(8.21)	(8.15)
		LAM	LAM	LAM	LAM	LAM
GDP PER WORKER AT DESTINATION	d_gdppwc	0.42	0.34	0.33	0.29	0.29
		(1.93)	(2.12)	(2.28)	(1.88)	(1.85)
		LAM	LAM	LAM	LAM	LAM
AREA OF DESTINATION	d_area	-0.04	-0.08	-0.07	-0.08	-0.08
		(-0.60)	(-1.29)	(-1.32)	(-1.52)	(-1.50)
		LAM	LAM	LAM	LAM	LAM
IMPEDANCES						
UTILITY	util_int3	0.11	0.16	0.15	0.16	0.16
		(4.36)	(5.95)	(5.66)	(5.62)	(5.60)
		LAM	LAM	LAM	LAM	LAM
BORDER IS CROSSED	cross	-0.63	-0.61	-0.50	-0.55	-0.55
	=====	(-1.94)	(-1.88)	(-1.52)	(-1.63)	(-1.63)
DATABASE						
CHANNEL SURVEY	ips_caa	-2.49	-1.92	-2.04	-1.95	-1.95
	=======	(-7.96)	(-5.67)	(-6.18)	(-5.75)	(-5.75)
REGRESSION CONSTANT	CONSTANT	-	-	-	-	-
		(0.98)	(5.48)	(6.07)	(4.84)	(4.76)
HETEROSKEDASTICITY STRUCTURE						
DELTA COEFFICIENTS						
DISTANCE BY CAR	dist			-0.26	-0.23	-0.23
				(-5.03)	(-4.40)	(-4.39)
				LAM	LAM	LAM

Table 11. (continued)

II. PARAMETERS and (COND. T-STATISTIC)		VARIANT =D 1.1	D 1.2	D 1.3	D 1.4	D 1.5
		FAMILY =LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2
		TYPE =LOG	BC8	BC8+ HE	BC8+ HE+AU	BC8+ HE+AU+PR
		PURPOSE =BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS
HETEROSKEDASTICITY STRUCTURE						
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]						
LAMBDA(Z)	dist			0.86	0.90	0.90
				[1.25]	[1.17]	[1.17]
				[-0.20]	[-0.14]	[-0.14]
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]						
LAMBDA(Y)	flow	0.00	0.03	0.07	0.06	0.06
		FIXED	[1.56]	[2.53]	[2.33]	[2.28]
			[-47.77]	[-35.13]	[-35.56]	[-34.90]
LAMBDA(X)	wpopxpsi3	0.00	1.59	1.76	1.62	1.62
		FIXED	[3.32]	[3.92]	[3.87]	[3.86]
			[1.23]	[1.70]	[1.48]	[1.48]
LAMBDA(X)	o_gdppwc	0.00	-1.15	-0.43	-0.59	-0.60
		FIXED	[-0.54]	[-0.28]	[-0.34]	[-0.34]
			[-0.90]	[-0.93]	[-0.90]	[-0.90]
LAMBDA(X)	o_area	0.00	0.75	0.85	0.92	0.92
		FIXED	[0.62]	[0.44]	[0.42]	[0.41]
			[-0.21]	[-0.08]	[-0.03]	[-0.03]
LAMBDA(X)	d_workpop	0.00	0.42	0.45	0.51	0.51
		FIXED	[1.74]	[1.80]	[2.06]	[2.06]
			[-2.37]	[-2.20]	[-2.00]	[-2.00]
LAMBDA(X)	d_gdppwc	0.00	0.78	0.80	0.87	0.87
		FIXED	[0.51]	[0.56]	[0.49]	[0.49]
			[-0.15]	[-0.14]	[-0.07]	[-0.07]
LAMBDA(X)	d_area	0.00	0.09	-0.07	-0.07	-0.07
		FIXED	[0.10]	[-0.08]	[-0.09]	[-0.09]
			[-0.95]	[-1.21]	[-1.35]	[-1.35]
LAMBDA(X)	util_int3	0.00	0.25	0.25	0.25	0.25
		FIXED	[3.67]	[3.76]	[3.67]	[3.66]
			[-11.08]	[-11.11]	[-11.24]	[-11.23]
SPATIAL AUTOCORRELATION: COND: [T-STATISTIC=0] / [T-STATISTIC=1]						
O and D: dist (100-300km)						
RHO (dist)					0.18	0.18
					(2.56)	(1.97)
PI (dist)					1.00	0.98
					FIXED	[1.91]
						[-0.04]

III. GENERAL STATISTICS	VARIANT =D 1.1	D 1.2	D 1.3	D 1.4	D 1.5
	FAMILY =LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2
	TYPE =LOG	BC8	BC8+ HE	BC8+ HE+AU	BC8+ HE+AU+PR
	PURPOSE =BUSINESS	BUSINESS	BUSINESS	BUSINESS	BUSINESS
LOG-LIKELIHOOD	-4890.33	-4868.30	-4858.08	-4852.16	-4852.16
SAMPLE: NUMBER OF PAIRS	544	544	544	544	544
NUMBER OF ESTIMATED PARAMETERS :					
- FIXED PART :					
. BETAS	10	10	10	10	10
. BOX-COX	0	8	8	8	8
. ASSOCIATED DUMMIES	0	0	0	0	0
- AUTOCORRELATION					
. RHO	0	0	0	1	1
. PI	0	0	0	0	1
- HETEROSKEDASTICITY :					
. DELTAS	0	0	1	1	1
. BOX-COX	0	0	1	1	1
. ASSOCIATED DUMMIES	0	0	0	0	0

4. Conclusion

In this paper, we perform four tasks.

- Firstly, we summarise the seminal fixed-form formulations of coupled two-stage models initiated 30 years ago in the context of Northeast Corridor Transportation Project of the U. S. Department of Transportation and show how the use of Box-Cox transformations made it possible a decade later both to unify the various model formulations and to challenge the credibility of model results obtained under fixed-form component assumptions, notably in gravity-type models of levels and in classical linear Logit-type choice models.

- Secondly, we note that, as soon as either component of two-stage models, the Level and the Split steps, are considered separately and can be effected either with aggregate or with disaggregate data, the four possible combinations define a structure, the Quasi-Direct Format, with four matching classes of algorithms.

- Thirdly, we summarise particular streams of improvement of three of these classes, all found in the TRIO program, Version 2.0, and concerned with the generalisation of so-called « root » models in different dimensions : in models of levels, ordinary least-squares procedures are generalised with respect to form of the fixed part and to heteroskedasticity and autocorrelation of the residuals ; in split models, the usual linear Logit model is generalised to non-linear forms that enrich the utility function and solve endemic parameter underidentification problems, and to the addition of choice captivity or modeler ignorance measures. We distinguish between background baseline pre-1996 developments and improvements under STEMM since 1996 : in models of levels, the algorithmic generalisations allow for tests of spatial autocorrelation of residuals with large samples ; in models of splits, numerous extensions of the Logit choice model are made possible with disaggregate data.

- Fourthly, we describe the first-ever multicountry application for passengers of the QDF format, obtained using the TRIO baseline algorithms : MAP-1 consists in a combination of disaggregate mode choice models and aggregate total demand models pertaining to three trip purposes. We also perform advanced tests of the algorithmic improvements effected under STEMM and demonstrate their usefulness on the business trip components of the MAP-1 multicountry application within the QDF structure.

Overall, the MAP-1 application and potentially improved estimates based on the demonstrated advanced tools establish that, with sufficient time and a more complete data base on passengers, networks and socio-economic variables, an EWAP (or Europe-Wide-Application for Passengers) extension of full geographic, modal and algorithmic coverage would successfully establish an operational reference model.

5. References

Algers, S. and M. Gaudry (1994), « Are Nonlinearity and Segmentation Substitutes in Logit Models ? », Unpublished manuscript, Royal Institute of Technology, Trafic and Transport Planning Department, Stockholm, 17 p. (in revision).

Armington, P., « A Theory of Demand for Products Distinguished by Place of Production », International Monetary Fund Staff Papers, 159-176, 1969.

Ben-Akiva, M., « Choice Models with Simple Choice Set Generating Processes », Working paper, Department of Civil Engineering, M.I.T., November 1977.

Berndt, E.R., Hall, B.H., Hall, R.E. and J.A. Hausman, « Estimation and Inference in Nonlinear Models », Annals of Economic and Social Measurement, 3, 653-665, 1974.

Blackburn, A.J., « Estimation of the Behavioral Model », in Studies in Travel Demand, Vol. V, Mathematica, March 1969.

Blum, U.C., Bolduc, D., and M. Gaudry, "From Correlation to Distributed Contiguities: a Family of AR-C-D Autocorrelation Processes". Publication CRT-734, Centre de recherche sur les transports, Université de Montréal, 1990, and Zeitschrift für Regionalwissenschaft 16/17, 99-128, 1995/1996.

Blum, U.C. and M. Gaudry, "The Impact of Social Security Contributions on Savings: an Analysis of German Households by Category Using Flexible Econometric Forms". Jahrbuch für Sozialwissenschaft 41, 2, 217-242, 1990.

Bolduc, D., « On the Estimation of Models with Generalized SAR(1) Processes on the Residuals of a Regression », Publication CRT-508, Centre de recherche sur les transports, Université de Montréal, 1987.

Bolduc, D., Dagenais, M.G. and M. Gaudry, "Spatially Autocorrelated Errors in Origin-Destination Models: A New Specification Applied to Aggregate Mode Choice", Transportation Research B 23, 5, 361-372, 1989.

Bordley, R.F., « Relaxing the Loyalty Condition in the Colombo/Morrison Model », Marketing Science, 8, 1, 100-103, 1989.

Bordley, R.F., « The Dogit Model is Applicable even without Perfectly Captive Buyers », Transportation Research B, 24, 4, 315-323, 1990.

Borenstein, S, and N.L. Rose, « Competition and Price Dispersion in the U.S. Airline Industry », Journal of Political Economy 102. 4, 653-683, 1994.

Box, G.P. and D.R. Cox, « An Analysis of Transformations », Journal of the Royal Statistical Society, Series B, 26, 211-243, 1964.

Cascetta, E., Nuzzolo, A., Russo, F. and A. Vitetta, « A New Route Choice Logit Model overcoming IIA Problems : Specification and Some Calibration Results for Interurban Networks », in J. B. Lesort (ed.) Transportation and Traffic Theory : Proceedings of the 13 th International Symposium on Transportation and Trafic Theory, Lyon, France, 15 p., 1996.

COST 305, « Data System for the Study of Demand for Interregional Passenger Transport », EUR 11443 EN, Commission of the European Communities, 1988.

Cox, D.R., Analysis of Binary Data, Methuen's Monographs on Applied Probability and Statistics, Methuen, London, 1970.

Crow, R., Young, K.H. and T. Cooley, « Models of Intercity Travel Demand. Part I. Theoretical Aspects », Mathematica, Inc., Princeton, New Jersey, for the Northeast Corridor Transportation Project, Department of Transportation, Washington, D.C., NTIS PB 201 206, 86 p., June 1971.

Dagenais, M.G. « Parameter Estimation in regression Models with Errors in the Variables and Autocorrelated Disturbances », Journal of Econometrics 64, 1-2, 145-163, 1994.

Dagenais, M. and M. Gaudry, "Can Aggregate Direct Travel Demand Models Work?", Proceedings of the World Conference on Transport Research, The Center for Transportation Studies, University of British Columbia, Vancouver, 1669-1676, 1986.

Dagenais, M., Gaudry, M. and T.C. Liem, "Urban Travel Demand: the Impact of Box-Cox Transformations with Nonspherical Residual Errors", Transportation Research B 21, 6, 443-477, 1987.

Davidson, R. and J.G. MacKinnon, «Transforming the Dependent Variable », Ch. 14 in Estimation and Inference in Econometrics , Oxford University Press, 1993.

Deepack, E. and R. Laferrière, « Demand Model Developments to Assess High Speed Passenger Train Markets on Windsor to Quebec City Corridor », Proceedings of the 20th Annual Meeting , Canadian Transportation Research Forum/Le Groupe de recherches sur les transports au Canada, 598-609, 1994.

Domencich, T. and D. McFadden, Urban Travel Demand : A Behavioral Analysis, North Holland, Amsterdam, 1975.

Ellis, R. H. and P.R. Rassam, « National Intercity Travel : Develoment and Implementation of a Demand Forecasting Framework » , U.S. Dept. of Transportation Contract T-8 542, Modification No. 1, National Technical Information Service PB 192 455, U. S. Dept. of Commerce, March 1970.

EUROSTAT, « Regions : Nomenclature of Territorial Units for Statistics—NUTS », 41 p., March 1992.

Fowkes, T. and J. Toner, « STEMM Ideal Freight Model Shell », Report 2.1, Ref. 14, Version 2.1, Institute for Transport Studies, The University of Leeds, 27 p., June 1998.

Fridström, L, « TRULS : an Econometric Model of Road Use, Accidents and their Severity », Institute of Transport Economics, Oslo, Preliminary draft, November 21, 1997.

Fridström, L. and A. Madslien, « A Stated Preference Analysis of Wholesalers' Freight Choice », Institute of Transport Economics, Oslo, 35 p., 1995.

Fry, T.R.L., « On the Use of some Burr Family Distributions in Econometrics », Ph.D. thesis, Faculty of Economic and Social Studies, University of Manchester, 1988.

Fry, T.R.L. and M.N. Harris, « The Size and Power Properties of Combining Choice Set Partition Tests for the IIA Property in the Logit Model », Department of Economics, Monash University, 1995.

Gaudry, M. "A Generalized Logit Mode Choice Model", Publication CRT-98, Centre de recherche sur les transports, et Cahier #7804, Département de sciences économiques, Université de Montréal, 20 p., 1978.

Gaudry, M., "Dogit and Logit Models of Travel Mode Choice in Montreal", Canadian Journal of Economics/Revue Canadienne d'Économique 13, 2, 268-279, 1980.

Gaudry, M. "The Inverse Power Transformation Logit and Dogit Mode Choice Models", Publication CRT-96, Centre de recherche sur les transports, Université de Montréal, April 1978, and Transportation Research B 15, 2, 97-103, 1981.

Gaudry, M., "Modèles agrégés et désagrégés à forme variable: résultats sur Montréal et Paris", Transports 304, 288-293, 1985.

Gaudry, M. "Asymmetric Shape and Variable Tail Thickness in Multinomial Probabilistic Responses to Significant Transport Service Level Changes », Publication CRT-893, Centre de recherche sur les transports, Université de Montréal, June 1989.

Gaudry, M., « FIQ : Fractional and Integer Quadratic Forms Estimated in TRIO », Manuscript, Centre de recherche sur les transports, Université de Montréal, 8 p., November 1996.

Gaudry, M. « Some Perspectives on the DRAG Approach and Family of National Road Safety Models », in Holst, H.v.., Nygren, A., and A. E. Andersson, eds., Transportation, Traffic Safety and Health III, 123-168, 1998.

Gaudry, M. « Are Fixed-Form Regression Models Ever Credible ? Some Evidence from Transportation Studies», in Gaudry, M. and R. R. Mayes, eds, Taking Stock of Air Liberalization, Ch. 12, 171-188, Kluwer Academic Press, 1999.

Gaudry, M. *et al.*, "Cur cum TRIO?", Publication CRT-901, Centre de recherche sur les transports, Université de Montréal, 20 pages, 1993, 1994, 1995, 1996, 1997.

Gaudry, M. and U.C. Blum, "An Example of Correlation Among Residuals in Directly Ordered Data", Economics Letters 26, 4,335-340, 1988.

Gaudry, M., Blum, U. and J. McCallum, "A First Gross Measure of Unexploited Single Market Integration Potential". In S. Urban, ed., Europe's Challenges, Gabler Verlag, 449-461, 1996.

Gaudry, M. and M. Dagenais, "The Dogit Model", Publication CRT-82, Centre de recherche sur les transports, Université de Montréal, Octobre 1977 and Transportation Research B 13, 2, 105-112, 1979.

Gaudry, M. and M. Dagenais, "The Generalized Dogit Mode Choice Model" Publication CRT-107, Centre de recherche sur les transports, et Cahier #7826, Département de sciences économiques, Université de Montréal, 20 p., 1978.

Gaudry, M. and M. Dagenais, "Heteroscedasticity and the Use of Box-Cox Transformations", Economics Letters 2, 3, 225-229, 1979.

Gaudry, M. and R. Laferrière, "The Box-Cox Transformation: Power Invariance and a New Interpretation", Economics Letters 30, 27-29, 1989.

Gaudry, M. and A. Le Leyzour, "Improving a Fragile Linear Logit Model Specified for High Speed Rail Demand Analysis in the Quebec-Windsor Corridor of Canada". Publication CRT-988 Centre de recherche sur les transports, Université de Montréal, forthcoming in Transportation Research E , 1994.

Gaudry, M., Mandel, B. and W. Rothengatter, "Introducing Spatial Competition through an Autoregressive Contiguous Distributed (AR-C-D) Process in Intercity Generation-Distribution Models within a Quasi-Direct Format (QDF), Publication CRT-971, Centre de recherche sur les transports, Université de Montréal, June 1994.

Gaudry, M., Jara-Diaz, S.R. and J. de D. Ortuzar, "Value of Time Sensitivity to Model Specification", Transportation Research B 23, 2, 151-158, 1989.

Gaudry, M. and M. J. Wills, "Estimating the Functional Form of Travel Demand Models", Publication CRT-63, Centre de recherche sur les transports, et Cahier 7702, Département de sciences économiques, Université de Montréal, juin 1977, and Transportation Research 12, 4, 257-289, 1978.

Gaudry, M. and M. Wills, "Testing the Dogit Model with Aggregate Time Series and Cross Sectional Travel Data", Transportation Research B 13, 2, 155-166, 1978.

Gillen, D., Harris, R. and T. Oum, « Evaluating Air Liberalization Agreements : An Integration of Demand Analysis and Trade Theory », Forthcoming in Gaudry, M. and R. Mayes, eds, Taking Stock of Air Liberalization, Kluwer Academic Press, 1998.

Goldberger, A.S., « Functional Form and Utility : A Review of Consumer Demand Theory », System Formulation, Methodology, and Policy Workshop Paper 6703, Social Systems Research Institute, University of Wisconsin, 122 p., October 1967.

Gunn, H.G. and J.J. Bates, « some Statistical Considerations in Fitting Travel Demand Models », International Conference on Travel Demand Models, Leeds, 1980.

Hensher, D.A. and Johnson, L.W., « Behavioural response and form of the representative component of the indirect utility function in travel mode choice », Regional Science and Urban Economics, 11, 559-572, 1981.

INSEE, Various questionnaires, « Enquête Transports 1993-1994 », or « Une enquête par sondage sur les transports en 1993-1994 », Institut National de la Statistique et des Etudes Économiques, mars 1993.

Kau, J.B. and C.F. Sirmans, « The Functional Form of the Gravity Model », International Regional Science Review, 4, 127-136, 1979.

Koppelman, F.S., « Nonlinear Utility Functions in Models of Travel Choice Behavior », The Transportation Center, Northwestern University, 37 p., May 1980.

Kraft, G., « Demand for Intercity Passenger Travel in the Washington-Boston Corridor », Part V, Northeast Corridor Project, U.S. Department of Transportation, 1963.

Laferrière, R., « Une Agrégation Nouvelle des Itinéraires de Transport Aérien (ANITA) », Publication CRT-574, Centre de recherche sur les transports, Université de Montréal, 106 p., Février 1988.

Laferrière, R., «The Air Network Itinerary and Trip Aggregation (ANITA) Model », Ch. 14 in Gaudry, M. and R. R. Mayes, eds, Taking Stock of Air Liberalization, Kluwer Academic Press, 223-228, 1999.

Laferrière, R. and M. Gaudry, "Testing the Linear Inverse Power Transformation Logit Mode Choice Model", Selected Proceedings of the Sixth World Conference on Transport Research, Vol. II, 739-750, 1993.

Last, J., « Reisezweckspezifische Modellierung von Verkehrsverflechtungen », Institut für Wirtschaftspolitik und Wirtschaftsforschung (IWW), and MKmetric GmbH, Karlsruhe, 132 p., 1998.

Leonardi, G., « The Structure of Random Utility Models in the Light of the Asymptotic Theory of Extremes », WP-82-91, International Institute for Applied Systems Analysis, 45 p., September 1982.

Leore, R., "Estimating O-D Travel from Link Traffic Counts: Can We Really Get There From Here?", Proceedings of the Canadian Transportation Research Forum's 31st Annual Conference, May 26-29, pp. 806-819, 1996a.

Leore, R. « An Economic Model of Inter-Urban Traffic on the Canadian Highway Network », Report TP 12800E, Economic Analysis, Transport Canada, 35 p., August 23, 1996b.

Lerman S. R. and J. J. Louvière, « On the Use of Functional Measurement to Identify the Functional Form of the Utility Expression in Travel Demand Models », Transportation Research Record 673, 78-86, 1978 .

Liem, T.C., Dagenais, M. and M. Gaudry, "LEVEL: The L-1.4 program for BC-GAUHESEQ regression —Box-Cox Generalized AUtoregressive HEteroskedastic Single EQuation models", Publication CRT-510, Centre de recherche sur les transports, Université de Montréal, 41 p., 1987, 1990, 1993.

Liem, T.C. and M. Gaudry, "PROBABILITY: The P-2 program for the Standard and Generalized BOX-COX LOGIT models with disaggregate data", Publication CRT-527, Centre de recherche sur les transports, Université de Montréal, 38 p., 1987, 1993.

Liem, T. C. and M. Gaudry, « PROBABILITY; The P-2 to P-6 programs for the Standard and Generalized BOX-COX-LOGIT and DOGIT and for the Linear and Box-Tukey INVERSE POWER TRANSFORMATION-LOGIT models with disaggregate data, Publication CRT-97-57, Centre de recherche sur les transports, Université de Montréal, 72 p., 1996 and 1997. Also Working Paper N° 9802, Bureau d'économie théorique et appliquée (BETA), Université Louis Pasteur, 1998a.

Liem, T.C. and M. Gaudry, "QDF: a Quasi-Direct Format used to combine total and mode choice results to obtain modal elasticities and diversion rates", Publication CRT-982, Centre de recherche sur les transports, Université de Montréal, June 1994, 1998. Also Working Paper N° 9812, Bureau d'économie théorique et appliquée (BETA), Université Louis Pasteur, 1998b.

Liem, T.C., M. Gaudry and U. Blum, « LEVEL: The L-2.0 program for BC-DAUHESEQ regressions —Box-Cox Directed AUtoregressive HEteroskedastic Single EQuation models, Publication CRT-972, Centre de recherche sur les transports, Université de Montréal, 23 p., 1994, 1997. Also Working Paper N° 9801, Bureau d'économie théorique et appliquée (BETA), Université Louis Pasteur, 1998.

Liem, T.C., Gaudry, M. and R. Laferrière "SHARE: The S-1 to S-5 programs for the Standard and Generalized BOX-C0X LOGIT and DOGIT and for the Linear and Box-Tukey INVERSE POWER TRANSFORMATION-LOGIT models with aggregate data", Publication CRT-899, Centre de recherche sur les transports, Université de Montréal, 70 pages, 1993, 1997.

Mandel, B., Schnellverkehr und ModalSplit, Nomos Verlag, Baden Baden, 1992.

Mandel, B., « The Interdependency of Airport Choice and Travel Demand », Ch. 13 in Gaudry, M. and R. Mayes, eds, Taking Stock of Air Liberalization, Kluwer Academic Press, 189-222, 1999.

Mandel, B., Gaudry, M. and W. Rothengatter, "A Disaggregate Box-Cox Logit Mode Choice of Intercity Passenger Travel in Germany and its Implications for High Speed Rail Demand Forecasts". The Annals of Regional Science 31, 2, 99-120, 1997.

McCarthy, P. S., « The Temporal Stability of Disaggregate Travel Demand Models », Transportation Research B, 16, 266- 276, 1982.

McFadden, D., « The Multinomial Logit Model when the Population Contains « Captive » Subpopulations », Unpublished Memorandum, 7 p., 30th September 1976.

McFadden, D., « Econometric Models of Probabilistic Choice », in Manski, C.F. and D. McFadden, eds, Structural Analysis of Discrete Data with Econometric Applications, M.I.T. Press, 1981.

McLynn, J.M., Goldman, A.J., Meyers, P.R. and R.H. Watkins, « Analysis of a Market Split Model », Journal of Research of the National Bureau of Standards, 72-B, #1, 1968.

McLynn, J.M., and T. Woronka, « A Family of Demand and Mode Split Models », Arthur Young and Co., April 1969.

Mishkin, F., « Does Correcting for Heteroskedasticity Help ? », Economics Letters 34, 351-356, 1990.

Moïsi, F., Raison, M. and L. Silman, « A Mode-Split Model with Multiple Transit Modes », Proceedings of the PTRC Summer Annual Meeting, 1981.

Montmarquette, C. and S. Mahseredjian, « Functional Forms and Educational Production Functions », Economics Letters, 19, 291-294, 1985.

Ord, C., « Estimation Methods for Models of Spatial Interaction », Journal of the American Statistical Association, 70, 349, 120-126, 1975.

Picard, G. and M. Gaudry, "Exploration of a Box-Cox Logit Model of Intercity Freight Mode Choice", Transportation Research E, 34, 1, 1-12, 1998.

Pong, S. « L'application du modèle probabiliste et de la transformation de Box-Cox au choix du mode de transport au Chili », Thèse de Maîtrise, Département de sciences économiques, Université de Montréal, 1991.

R.A.T.P. et Cambridge Systematics, « Etudes de politiques de transport en région Ile-de-France. Mise au point et utilisation de modèles désagrégés de choix modal ». R.A.T.P., Direction des études générales, Service GL, 53ter Quai des Grands Augustins, Paris 6ème, juin 1982.

Quandt, R.E. and Baumol, W.J., « « The Demand for Abstract Transport Modes : Theory and Measurement », Journal of Regional Science, 6, 2, 13-26, 1966.

Rassam, P. R., Ellis, R.H. and J.C. Bennett, « The N-Dimensional Logit Model : Development and Application », Peat Marwick and Mitchell & Co, Washington, D.C., 42p., 1970. Also in Highway Research Record, 369, 135-147, 1971.

Restle, F., Psychology of Judgment and Choice , John Wiley and Sons, New York, 1961.

Samuelson, L., « On the Independence from Irrelevant Alternatives in Probabilistic Choice Models », Journal of Economic Theory, 35, 2, 376-389, 1985.

Schlesselman, J., « Power Families : A Note on the Box and Cox Transformation », Journal of the Royal Statistical Society, Series B, 33, 307-311, 1971.

Schnetzler, B., « Modèles stochastiques de la décision. L'erreur de prédiction», Rapport INRETS n° 207, Institut National de Recherche sur les Transports et leur Sécurité, 71 p., mars 1996.

Schnetzler, B., « Modèles stochastiques du choix discret - I et II », Unpublished manuscript, 70 p., Submitted to Annales d'économie et de Statistique, 1998.

Small, K. E. « Review of J.D. Westley 'Planning the Location of Urban-Suburban Rail Lines : an Application of Cost-Benefit and Optimal Path Analysis' », Transportation Research A, 15, 1, 11-112, 1981.

Spanos, A., « Error Autocorrelation Revisited: the AR(1) Case », Econometric Review 6, 2, 285-294, 1987/1988.

Spitzer, J. J., « Variance Estimates in Models with the Box-Cox Transformation : Implications for Estimation and Hypothesis Testing », The Review of Economics and Statistics 66, 645-652, 1984.

Tardiff, T.J., « The Use of Alternative Specific Constants in Choice Modelling », UCI-ITS-SP-78-6, Institute of Transportation Studies, University of California, Irvine,28 p., December 1978.

Tegnér, G., Loncar-Lucassi, V., Nilsson, C. and I. Holmberg, « The Demand for Public Transport Trips in Stockholm County –a Two-Stage Aggregate, Non-Linear Time-Series Model », Transek AB, Stockholm, 47 p., 30th April 1998.

Theil, H., « The Theory of Rational Random Behaviour and its Application to Demand Analysis », European Economic Review, 6, 217-226, 1975.

Train, K.E. and D. McFadden, « The Goods/Leisure Trade-Off and Disaggregate Work Trip Mode Choice Models », Transportation Research 12, 5, 349-353, 1978.

Truong, T. P., « A Theoretical Analysis of Choice Demand and Time Allocation : with Reference to Travel », Ph. D. Thesis, School of Economics and Financial Studies, Macquarie University, 282 p., December 1981.

Warner, S.L., Stochastic Choice of Mode in Urban Travel : A Study in Binary Choice, Northwestern University Press, Evanston, Ill., 1962.

Westley, J.D., « The Demand for Urban Rail Transportation », Journal of the American Statistical Association, 74, 576-587, 1979.

Williams, H.C.W.L., « On the Formation of Travel Demand Models and Economic Evaluation Measures of User Benefit ». Environment and Planning, 9a(3), 285-344, 1977.

Williams, H.C.W.L. and J. D. Ortuzar, « Behavioural Theories of Dispersion and Mis-specification of Travel Demand Models », Transportation Research B, 16, 3, 167-219, 1982.

Wills, M. J. , « A Flexible Gravity-Opportunities Model for Trip Distribution », Transportation Research ,Vol. 20B, No. 2, pp. 89-111, 1986.

Wilson, A., « A Statistical Theory of Spatial Distribution Models », Transportation Research, 1, 253-269, 1967.

Young, K. H., « An Abstract Mode Approach to the Demand for Travel », Transportation Research, 3, 443-461, 1969.

Zarembka, P., « Transformation of Variables in Econometrics », Ch. 3 in P. Zarembka, ed., Frontiers in Econometrics, 1974.

Appendix 1. Detailed Results of Table 8.

Section I of this table on this and the next 2 pages contains, for each variable, the elements (iv) and (vi) already found in Table 8, but also, in turn:

- **(i) the regression coefficient associated with this variable;**
- **(ii) the own elasticity evaluated at the means, according to Eq. (43) ;**
- **(iii) the marginal rate of substitution (value of time) calculated as ratio of the partial derivatives of this variable and the own modal cost variable —this ratio is equal to one for the cost variable itself;**
- **(iv) the weighted aggregate elasticity, according to Eq. (44) ;**
- **(v) the own weighted aggregate change in probability, i.e. Eq. (40) aggregated as in Eq. (44);**
- **(vi) the asymptotic t-statistics of the regression coefficient of this variable computed conditionally upon the estimated value of the BC transformations (in Eq. (17) and Eq. (18-B)) ;**
- **(vii) the own and cross change in probability, i.e. Eq. (40) aggregated as in Eq. (43)..**

And we note that the three modes are designated as modes 1 (air), 2 (train) and 4 (car) when cross-values are listed for (viii).

MAP-1 Reference Asymmetric Response Mode Choice Models (F. Heinitz, July 1998)
LOGIT Family
Standard Linear and Standard BOX-COX Types (PROBABILITY P-2 Algorithm)
Choice Probability for Three Modes: Business, Private and Vacation Trips (CAA/IPS 1996 data)

```
===============================================================================================
I.  BETA                        VARIANT = MAP 1.1  MAP 1.2  MAP 2.1  MAP 2.2  MAP 3.1  MAP 3.2
    ELASTICITIES AT MEAN         FAMILY = LOGIT    LOGIT    LOGIT    LOGIT    LOGIT    LOGIT
    MRS AT MEAN: Cost              TYPE = linlogit bc1logit linlogit bc1logit linlogit bc1logit
    W.A. ELASTICITIES           PURPOSE = BUSINESS BUSINESS PRIVATE  PRIVATE  VACATION VACATION
    W.A. CHANGE IN P(I)
   (COND. T-STATISTIC)
    CHANGE IN P(I) AT MEAN
===============================================================================================
ALTERNATIVE   1 : AIR
=====================
-----------------
NETWORK VARIABLES
-----------------
 TOTAL GROUP COST [AIR] TXCostA           -.10E-02 -.81E+02 -.53E-03 -.67E+01 -.68E-03 -.70E-03
                                             -.051    -.077    -.038    -.080    -.083    -.078
                         (1)               .10E+01  .10E+01  .10E+01  .10E+01  .10E+01  .10E+01
                                             -.067    -.089    -.048    -.088    -.094    -.095
                                             -.058    -.077    -.040    -.073    -.059    -.061
                                          (-18.61) (-20.30) (-10.01) (-12.74) (-23.78) (-24.32)
                                        1    -.048    -.072    -.034    -.071    -.070    -.067
                                        2     .027     .039     .012     .024     .032     .031
                                        4     .020     .033     .022     .047     .038     .036
                                          L  1(GE) L  1(GE) L  1(GE) L  1(GE) L  1(GE)     (GE)

  TOTAL                  TTTimeA          -.20E-02 -.73E+02 -.97E-03 -.90E+01
  TRAVEL TIME [AIR]                          -.037    -.126    -.043    -.126
                         (1)               .19E+01  .43E+01  .18E+01  .26E+01
                                             -.050    -.144    -.051    -.141
                                             -.044    -.126    -.044    -.118
                                          (-14.25) (-23.03)  (-9.47) (-17.77)
                                        1    -.035    -.118    -.039    -.112
                                        2     .020     .065     .013     .038
                                        4     .015     .054     .025     .074
                                          L  1(GE) L  1(GE) L  1(GE) L  1(GE)

  TOTAL                  TOoVTimA         -.29E-02 -.68E-02 -.14E-02 -.28E-02
  OUT-OF-VEHICLE [AIR]                       -.012    -.030    -.010    -.024
                         (1)               .29E+01  .48E+01  .27E+01  .30E+01
                                             -.015    -.033    -.013    -.025
                                             -.013    -.029    -.011    -.021
                                           (-2.35)  (-6.59)  (-1.44)  (-2.98)
                                        1    -.011    -.028    -.009    -.021
                                        2     .006     .015     .003     .007
                                        4     .005     .013     .006     .014
                                              (GE)     (GE)     (GE)     (GE)
-----------------------------------------------------------------------------------------------

  REGRESSION CONSTANT    CONSTANT          .32E+01  .29E+01  .26E+01  .26E+01 -.59E+00 -.25E+01
                                                -        -        -        -        -        -
                         (1)                    -        -        -        -        -        -
                                                -        -        -        -        -        -
                                                -        -        -        -        -        -
                                           (34.31)  (31.05)  (35.53)  (35.38) (-16.03) (-41.14)
                                                -        -        -        -        -        -
```

Appendix 1. Detailed Results of Table 8. (continued)

```
=============================================================================================
I.  BETA                           VARIANT = MAP 1.1  MAP 1.2  MAP 2.1  MAP 2.2  MAP 3.1  MAP 3.2
    ELASTICITIES AT MEAN            FAMILY = LOGIT    LOGIT    LOGIT    LOGIT    LOGIT    LOGIT
    MRS AT MEAN: Cost                 TYPE = linlogit bc1logit linlogit bc1logit linlogit bc1logit
    W.A. ELASTICITIES              PURPOSE = BUSINESS BUSINESS PRIVATE  PRIVATE  VACATION VACATION
    W.A. CHANGE IN P(I)
   (COND. T-STATISTIC)
    CHANGE IN P(I) AT MEAN
=============================================================================================
ALTERNATIVE   2 : RAIL
======================
-----------------
NETWORK VARIABLES
-----------------

  TOTAL GROUP         TXCostR          -.10E-02 -.81E+02 -.53E-03 -.67E+01 -.68E-03 -.70E-03
  COST [RAIL]                              -.662   -1.378    -.193    -.890    -.285    -.292
                      (2)               .10E+01  .10E+01  .10E+01  .10E+01  .10E+01  .10E+01
                                           -.568   -1.396    -.151    -.877    -.200    -.203
                                           -.043    -.182    -.017    -.130    -.040    -.040
                                       (-18.61) (-20.30) (-10.01) (-12.74) (-23.78) (-24.32)
                                     1      .022     .046     .006     .030     .019     .018
                                     2     -.022    -.047    -.006    -.033    -.021    -.020
                                     4      .001     .001     .000     .002     .002     .002
                                       L  1(GE) L  1(GE) L  1(GE) L  1(GE) L  1(GE)     (GE)

  TOTAL               TTTimeR          -.20E-02 -.73E+02 -.97E-03 -.90E+01 -.31E-02 -.70E-01
  TRAVEL TIME [RAIL]                      -1.331   -1.212    -.821    -.897   -3.048   -2.663
                      (2)               .19E+01  .85E+00  .18E+01  .44E+00  .45E+01  .38E+01
                                           -.995   -1.316    -.542    -.910   -1.552   -1.634
                                           -.069    -.156    -.053    -.137    -.260    -.313
                                       (-14.25) (-23.03)  (-9.47) (-17.77) (-68.32) (-70.66)
                                     1      .043     .040     .025     .031     .201     .167
                                     2     -.045    -.041    -.027    -.033    -.222    -.182
                                     4      .001     .001     .002     .002     .021     .015
                                       L  1(GE) L  1(GE) L  1(GE) L  1(GE) L  1(GE) L  1(GE)

  TOTAL               TOoVTimR         -.29E-02 -.68E-02 -.14E-02 -.28E-02
  OUT-OF-VEHICLE [RAIL] --------           -.102    -.238    -.052    -.104
                      (2)               .29E+01  .32E+01  .27E+01  .12E+01
                                           -.091    -.206    -.043    -.087
                                           -.007    -.027    -.005    -.012
                                         (-2.35)  (-6.59)  (-1.44)  (-2.98)
                                     1      .003     .008     .002     .004
                                     2     -.003    -.008    -.002    -.004
                                     4      .000     .000     .000     .000
                                            (GE)     (GE)     (GE)     (GE)

  TOTAL NUMBER OF     TUmstR           -.36E+00 -.37E+00 -.69E+00 -.59E+00 -.61E-01 -.62E-01
  TRANSFERS [RAIL]                        -1.555   -1.584   -3.269   -2.798    -.295    -.304
                      (2)               .36E+03  .18E+03  .13E+04  .24E+03  .89E+02  .89E+02
                                          -1.105   -1.063   -2.043   -1.731    -.173    -.178
                                           -.074    -.096    -.201    -.182    -.032    -.032
                                       (-14.25) (-15.93  (-24.68) (-22.00)  (-5.10)  (-5.29)
                                     1      .051     .053     .101     .096     .020     .019
                                     2     -.052    -.054    -.108    -.103    -.022    -.021
                                     4      .001     .002     .007     .008     .002     .002
                                            (SP)     (SP)     (SP)     (SP)     (SP)     (SP)

---------------------------------------------------------------------------------------------

  REGRESSION CONSTANT CONSTANT          .19E+01  .19E+01  .28E+01  .23E+01  .35E+00  .36E+00
                                            -        -        -        -        -        -
                      (2)                   -        -        -        -        -        -
                                            -        -        -        -        -        -
                                            -        -        -        -        -        -
                                         (19.09)  (18.43)  (26.97)  (19.45)   (7.17)   (7.55)
                                            -        -        -        -        -        -
```

Appendix 1. Detailed Results of Table 8. (continued)

```
=================================================================================================
I.  BETA                        VARIANT = MAP 1.1  MAP 1.2  MAP 2.1  MAP 2.2  MAP 3.1  MAP 3.2
    ELASTICITIES AT MEAN         FAMILY = LOGIT    LOGIT    LOGIT    LOGIT    LOGIT    LOGIT
    MRS AT MEAN: Cost              TYPE = linlogit bc1logit linlogit bc1logit linlogit bc1logit
    W.A. ELASTICITIES           PURPOSE = BUSINESS BUSINESS PRIVATE  PRIVATE  VACATION VACATION
    W.A. CHANGE IN P(I)
   (COND. T-STATISTIC)
    CHANGE IN P(I) AT MEAN
=================================================================================================
ALTERNATIVE   4 : CAR
=====================
-----------------
NETWORK VARIABLES
-----------------
  TOTAL COST [CAR]       TCostC          -.10E-02 -.81E+02 -.53E-03 -.67E+01 -.68E-03 -.70E-03
                                             -.837   -1.205    -.223    -.809    -.317    -.327
                         (4)               .10E+01  .10E+01  .10E+01  .10E+01  .10E+01  .10E+01
                                             -.758   -1.250    -.221    -.810    -.220    -.223
                                             -.039    -.084    -.015    -.064    -.041    -.041
                                         (-18.61) (-20.30) (-10.01) (-12.74) (-23.78) (-24.32)
                                       1      .020     .033     .013     .056     .025     .024
                                       2      .001     .001     .000     .002     .002     .002
                                       4     -.021    -.034    -.014    -.058    -.027    -.025
                                         L  1(GE) L  1(GE) L  1(GE) L  1(GE) L  1(GE)     (GE)

  TOTAL                  TTTimeC         -.20E-02 -.73E+02 -.97E-03 -.90E+01 -.31E-02 -.70E-01
  TRAVEL TIME [CAR]                         -1.246   -1.277    -.725    -.892   -2.795   -2.536
                         (4)               .19E+01  .14E+01  .18E+01  .62E+00  .45E+01  .39E+01
                                            -1.120   -1.339    -.719    -.910   -1.595   -1.660
                                             -.058    -.088    -.049    -.071    -.258    -.304
                                         (-14.25) (-23.03)  (-9.47) (-17.77) (-68.32) (-70.66)
                                       1      .030     .035     .043     .062     .221     .183
                                       2      .001     .001     .002     .003     .019     .015
                                       4     -.031    -.036    -.045    -.064    -.240    -.197
                                         L  1(GE) L  1(GE) L  1(GE) L  1(GE) L  1(GE) L  1(GE)

  TOTAL                  TOoVTimC        -.29E-02 -.68E-02 -.14E-02 -.28E-02
  OUT-OF-VEHICLE [CAR]   --------            -.028    -.066    -.013    -.027
                         (4)               .29E+01  .46E+01  .27E+01  .15E+01
                                             -.028    -.061    -.013    -.023
                                             -.001    -.004    -.001    -.002
                                           (-2.35)  (-6.59)  (-1.44)  (-2.98)
                                       1      .001     .002     .001     .002
                                       2      .000     .000     .000     .000
                                       4     -.001    -.002    -.001    -.002
                                              (GE)     (GE)     (GE)     (GE)
```

Appendix 1. Detailed Results of Table 8. (continued)

II. PARAMETERS UNCOND. [T-STATISTIC=0] UNCOND. [T-STATISTIC=1]							
	VARIANT =	MAP 1.1	MAP 1.2	MAP 2.1	MAP 2.2	MAP 3.1	MAP 3.2
	FAMILY =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
	TYPE =	linlogit	bc1logit	linlogit	bc1logit	linlogit	bc1logit
	PURPOSE =	BUSINESS	BUSINESS	PRIVATE	PRIVATE	VACATION	VACATION
BOX-COX TRANSFORMATIONS							
LAMBDA(X) - GROUP 1 LAM 1		1.000	-.620	1.000	-.334	1.000	.531
		FIXED	[-13.39]	FIXED	[-7.47]	FIXED	[10.79]
			[-34.99]		[-29.84]		[-9.51]

III. GENERAL STATISTICS							
	VARIANT =	MAP 1.1	MAP 1.2	MAP 2.1	MAP 2.2	MAP 3.1	MAP 3.2
	FAMILY =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
	TYPE =	linlogit	bc1logit	linlogit	bc1logit	linlogit	bc1logit
	PURPOSE =	BUSINESS	BUSINESS	PRIVATE	PRIVATE	VACATION	VACATION
LOG-LIKELIHOOD -FINAL VALUE		-6731.15	-6246.86	-7288.48	-7117.35	-20343.0	-20299.7
-INITIAL VALUE		-7286.77	-24563.9	-7926.74	-7926.74	-25422.2	-25422.2
-WITH CONSTANTS ONLY		-7286.77	-7286.77	-7926.74	-7926.74	-25422.2	-25422.2
-RATIO TEST		1111.242	2079.823	1276.522	1618.773	10158.46	10245.05
RHO-SQUARED		.076	.143	.081	.102	.200	.201
RHO-SQUARED BAR -AKAIKE		.075	.142	.080	.101	.200	.201
-HOROWITZ		.076	.142	.080	.102	.200	.201
-HENSHER AND JOHNSON		.076	.143	.080	.102	.200	.201
PER CENT RIGHT		92.066	92.187	87.665	87.607	76.237	77.500
SAMPLE -NUMBER OF ALTERNATIVES		3	3	3	3	3	3
-NUMBER OF OBSERVATIONS		22579	22579	17477	17477	37512	37512
-FIRST OBSERVATION		1	1	1	1	1	1
-LAST OBSERVATION		22579	22579	17477	17477	37512	37512
-AVAILABLE OBSERVATIONS IN VT		22359	22359	17308	17308	35004	35004
-AVAIL OBS IN 1 AIR		22579	22579	17477	17477	37512	37512
-AVAIL OBS IN 2 RAIL		22579	22579	17477	17477	37512	37512
-AVAIL OBS IN 4 CAR		22579	22579	17477	17477	37512	37512
NUMBER OF ESTIMATED PARAMETERS							
-BETAS .VARIABLES		4	4	4	4	3	3
.CONSTANTS		2	2	2	2	2	2
.ASSOCIATED DUMMIES		0	0	0	0	0	0
-BOX-COX TRANSFORMATIONS		0	1	0	1	0	1

Appendix 2. Background Business Generation-Distribution Models (J. Last, July 1998)
BC-GAUHESEQ Family (LEVEL 1.4 algorithm)
Total Flow by all Modes; Business Trips (CAA/IPS 1991 and SES 1993-1994 data)

I. BETA / ELASTICITY E(y) (EP) / PARTIAL DER. E(y) (EP) / (COND. T-STATISTIC)	TYPE = / VARIANT = / VERSION =	LEVEL-1 busM 1	LEVEL-1 busM 2	LEVEL-1 busM 3	LEVEL-1 busM 4	LEVEL-1 busM 5
Origin characteristics						
Working population weighted by a Psi index based on delta values	wpopxpsi3	.311836E+00	.305286E+00	.262024E+00	.246983E+00	.255095E+00
		.312	.305	.308	.323	.325
		.110830E-02	.102930E-02	.102989E-02	.105568E-02	.169985E-02
		(13.82)	(14.59)	(15.47)	(17.00)	(17.65)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
Area Orig	o_area	-.520104E-01	-.555140E-01	-.602697E-01	-.672937E-01	-.633931E-01
		-.052	-.056	-.059	-.065	-.060
		-.550463E-01	-.557372E-01	-.588366E-01	-.636480E-01	-.936692E-01
		(-2.55)	(-2.65)	(-2.90)	(-3.43)	(-3.23)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
GDP per (working) capita Orig	o_gdppwc	.552748E+00	.598353E+00	.502023E+00	.665664E+00	.698473E+00
		.553	.598	.577	.568	.550
		.754532E+05	.774841E+05	.741510E+05	.713228E+05	.110318E+06
		(8.12)	(8.46)	(8.46)	(9.50)	(9.43)
		LAM 3	LAM 3	LAM 3	LAM 3	LAM 3
Destination characteristics						
Area Dest	d_area	-.887041E-01	-.900633E-01	-.971156E-01	-.106079E+00	-.101651E+00
		-.089	-.090	-.095	-.103	-.097
		-.899100E-01	-.865997E-01	-.909204E-01	-.963039E-01	-.144166E+00
		(-4.70)	(-4.63)	(-5.13)	(-5.95)	(-5.69)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
GDP per (working) capita Dest	d_gdppwc	.484279E+00	.536491E+00	.448975E+00	.548976E+00	.592593E+00
		.484	.536	.516	.468	.466
		.658271E+05	.691792E+05	.660026E+05	.585511E+05	.931737E+05
		(7.73)	(8.47)	(8.72)	(9.10)	(9.27)
		LAM 3	LAM 3	LAM 3	LAM 3	LAM 3
Working population Dest	d_workpop	.428184E+00	.422373E+00	.356128E+00	.333828E+00	.345506E+00
		.428	.422	.420	.439	.443
		.135729E-02	.127011E-02	.125299E-02	.128023E-02	.206559E-02
		(18.30)	(19.69)	(20.78)	(22.98)	(23.71)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
Impedances						
Cross-border journey (Dummy)	cross =====	-.637238E+00	-.555903E+00	-.749669E+00	-.860753E+00	-.130062E+01
		-.637	-.556	-.567	-.547	-.811
		-.231000E+04	-.191167E+04	-.193416E+04	-.182433E+04	-.431926E+04
		(-5.49)	(-3.75)	(-4.28)	(-4.07)	(-6.46)
Travel time (car)	time	-.335671E+00	-.677057E+02	-.108753E+03	-.235702E+03	
		-.336	-.239	-.234	-.207	
		-.191636E+01	-.129490E+01	-.125620E+01	-.108553E+01	
		(-8.78)	(-9.92)	(-11.09)	(-10.02)	
		LAM	LAM	LAM	LAM	
Utility index [exp(util_int)]	util_int3					.109742E+01
						.226
						.337982E+05
						(12.50)
						LAM
Database and associated dummy						
Screenline survey (Cross-channel)	ips_caa =======	-.275841E+01	-.270247E+01	-.346732E+01	-.418035E+01	-.322404E+01
		-2.758	-2.702	-2.622	-2.658	-2.010
		-.999926E+04	-.929343E+04	-.894573E+04	-.886009E+04	-.107068E+05
		(-25.23)	(-19.39)	(-21.04)	(-21.52)	(-15.24)
Working population weighted by a Psi index based on delta values	wpopxpsi3 =========	-.340235E+01	-.332037E+01	-.330025E+01	-.361556E+01	-.395352E+01
		-3.402	-3.320	-2.495	-2.299	-2.464
		-.123336E+05	-.114183E+05	-.851469E+04	-.766303E+04	-.131293E+05
		(-5.48)	(-3.35)	(-2.84)	(-2.31)	(-2.49)

Appendix 2. (Continued)

		1	2	3	4	5
REGRESSION CONSTANT	CONSTANT	.103503E+02 - - (15.19)	.856256E+02 - - (11.13)	.129391E+03 - - (12.14)	.242481E+03 - - (10.62)	.144098E+02 - - (8.88)

HETEROSKEDASTICITY STRUCTURE

DELTA COEFFICIENTS

		1	2	3	4	5
Travel distance (car)	dist				-.325565E-02 -.192 -.728983E+00 (-11.01) LAM	-.155296E-02 -.173 -.104852E+01 (-10.39) LAM

II. PARAMETERS and (COND. T-STATISTIC)	TYPE = VARIANT = VERSION =	LEVEL-1 busM 1	LEVEL-1 busM 2	LEVEL-1 busM 3	LEVEL-1 busM 4	LEVEL-1 busM 5
HETEROSKEDASTICITY STRUCTURE						
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]						
LAMBDA(Z)	dist				.683 [2.39] [-1.11]	.783 [2.47] [-.68]
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]						
LAMBDA(Y) - GROUP 1	LAM 1	.000 FIXED	.000 FIXED	.032 [5.24] [-159.19]	.052 [7.03] [-127.76]	.052 [7.06] [-129.29]
LAMBDA(X)	time	.000 FIXED	-.875 [-5.25] [-11.26]	-.908 [-5.67] [-11.90]	-1.020 [-5.81] [-11.50]	
LAMBDA(X)	util_int3					.333 [9.56] [-19.20]
LAMBDA(X) - GROUP 1	LAM 1	.000 FIXED	.000 FIXED	.032 [5.24] [-159.19]	.052 [7.03] [-127.76]	.052 [7.06] [-129.29]
LAMBDA(X) - GROUP 3	LAM 3	.000 FIXED	.000 FIXED	-.115 [-.66] [-6.36]	-.081 [-.41] [-5.44]	-.064 [-.33] [-5.41]

III.GENERAL STATISTICS	TYPE = VARIANT = VERSION =	LEVEL-1 busM 1	LEVEL-1 busM 2	LEVEL-1 busM 3	LEVEL-1 busM 4	LEVEL-1 busM 5
LOG-LIKELIHOOD		-50846.203	-50826.210	-50809.285	-50762.429	-50737.562
SAMPLE : - NUMBER OF OBSERVATIONS		5711	5711	5711	5711	5711
- FIRST OBSERVATION		1	1	1	1	1
- LAST OBSERVATION		5711	5711	5711	5711	5711
NUMBER OF ESTIMATED PARAMETERS :						
- FIXED PART :						
. BETAS		11	11	11	11	11
. BOX-COX		0	1	3	3	3
. ASSOCIATED DUMMIES		1	1	1	1	1
- HETEROSKEDASTICITY :						
. DELTAS		0	0	0	1	1
. BOX-COX		0	0	0	1	1
. ASSOCIATED DUMMIES		0	0	0	0	0

Appendix 3. Background Private Generation-Distribution Models (J. Last, July 1998)
BC-GAUHESEQ Family (LEVEL 1.4 algorithm)
Total Flow by all Modes; Private Trips (CAA/IPS 1991 and SES 1993-1994 data)

I. BETA ELASTICITY E(y) (EP) PARTIAL DER. E(y) (EP) (COND. T-STATISTIC)	TYPE = VARIANT = VERSION =	LEVEL-1 prvD 1	LEVEL-1 prvD 2	LEVEL-1 prvD 3	LEVEL-1 prvD 4	LEVEL-1 prvD 5
Origin characteristics						
Population	popxpsi3	.322915E+00	.309875E+00	.281771E+00	.287757E+00	.302248E+00
weighted by a		.323	.310	.319	.323	.336
Psi index based		.204749E-02	.174459E-02	.176023E-02	.171876E-02	.413083E-02
on delta values		(18.73)	(18.13)	(19.52)	(19.83)	(21.04)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
GDP per capita Orig	o_gdppc	-.878360E-01	.222751E+01	.260755E+01	.224628E+01	.429734E+01
		-.001	.038	.034	.030	.054
		-.649793E+03	.146317E+05	.128639E+05	.109161E+05	.459892E+05
		(-.05)	(1.14)	(1.07)	(.96)	(1.76)
Destination characteristics						
Population Dest	d_pop	.487552E+00	.485813E+00	.418477E+00	.417997E+00	.446403E+00
		.488	.486	.475	.471	.497
		.274819E-02	.243146E-02	.233165E-02	.222625E-02	.544191E-02
		(24.59)	(25.67)	(26.34)	(26.35)	(28.10)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
Seaside at Dest	seaside	.124512E+00	.126632E+00	.166872E+00	.172727E+00	.168569E+00
but not in Orig	=======	.125	.127	.128	.135	.126
		.921114E+03	.831797E+03	.823233E+03	.839392E+03	.180399E+04
		(3.57)	(3.69)	(3.89)	(4.07)	(3.92)
Impedances						
Cross-border journey	cross	-.509541E+00	-.497821E+00	-.685547E+00	-.661675E+00	-.522007E+00
	=====	-.510	-.498	-.525	-.518	-.389
		-.376948E+04	-.327001E+04	-.338203E+04	-.321550E+04	-.558640E+04
		(-4.57)	(-4.05)	(-4.70)	(-4.78)	(-3.63)
Common	lang2	.287151E+00	.197315E+00	.246007E+00	.230577E+00	.174848E+00
language in Dest	=====	.287	.197	.188	.181	.130
and Orig and		.212428E+04	.129609E+04	.121364E+04	.112052E+04	.187118E+04
Cross-border		(1.39)	(.91)	(.96)	(.94)	(.68)
Travel time (car)	time	-.722896E+00	-.221918E+02	-.327290E+02	-.255252E+02	
		-.723	-.600	-.594	-.633	
		-.806174E+01	-.594344E+01	-.576205E+01	-.592327E+01	
		(-22.40)	(-27.32)	(-30.00)	(-30.18)	
		LAM 2	LAM 2	LAM 2	LAM 2	
Utility index	util_int3					.325406E+01
[exp(util_int)]						.529
						.930952E+06
						(26.99)
						LAM
Database						
Screenline survey	ips_caa	-.262970E+01	-.246057E+01	-.304047E+01	-.296898E+01	-.268034E+01
	=======	-2.630	-2.461	-2.329	-2.324	-1.997
		-.194540E+05	-.161626E+05	-.149997E+05	-.144282E+05	-.286844E+05
		(-22.64)	(-18.98)	(-19.67)	(-20.17)	(-16.74)
Outlier record # 1642	rec1642	.320280E+01	.332305E+01	.438194E+01	.430741E+01	.421692E+01
	=======	3.203	3.323	3.357	3.372	3.142
		.236937E+05	.218279E+05	.216176E+05	.209325E+05	.451286E+05
		(2.40)	(.00)	(.00)	(.00)	(.00)

Appendix 3. (Continued)

```
-------------------------------------------------------------------------------------------------
  REGRESSION CONSTANT    CONSTANT      .356559E+01  .376616E+02  .549400E+02  .459324E+02  .730371E+01
                                                -            -            -            -            -
                                                -            -            -            -            -
                                           (9.41)      (26.55)      (30.03)      (29.96)      (17.53)
 ----------------------------
 HETEROSKEDASTICITY STRUCTURE
 ----------------------------
  DELTA COEFFICIENTS
  ------------------
  Travel distance (car)  dist                                                .105182E-12
                                                                                    .022
                                                                             .145728E+00
                                                                                  (5.75)
                                                                                     LAM

=================================================================================================
 II. PARAMETERS                TYPE =  LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1
     and                    VARIANT =    prvD         prvD         prvD         prvD         prvD
    (COND. T-STATISTIC)     VERSION =      1            2            3            4            5
==================================================================================================
 ----------------------------
 HETEROSKEDASTICITY STRUCTURE
 ----------------------------
  BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]
  ------------------------------------------------------------------
  LAMBDA(Z)              dist                                                      3.858
                                                                                  [2.74]
                                                                                  [2.03]

 ------------------------------------------------------------------
 BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]
 ------------------------------------------------------------------
  LAMBDA(Y) - GROUP  1   LAM  1               .000         .000         .028         .026         .029
                                             FIXED        FIXED       [6.14]       [5.69]       [6.24]
                                                                   [-213.59]    [-214.70]    [-212.32]

  LAMBDA(X)              util_int3                                                              .317
                                                                                             [16.17]
                                                                                             [-34.90]

  LAMBDA(X) - GROUP  1   LAM  1               .000         .000         .028         .026         .029
                                             FIXED        FIXED       [6.14]       [5.69]       [6.24]
                                                                   [-213.59]    [-214.70]    [-212.32]

  LAMBDA(X) - GROUP  2   LAM  2               .000        -.556        -.576        -.531
                                             FIXED     [-13.21]     [-14.90]     [-12.56]
                                                       [-36.99]     [-40.77]     [-36.21]

==================================================================================================
 III.GENERAL STATISTICS        TYPE =  LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1
                            VARIANT =    prvD         prvD         prvD         prvD         prvD
                            VERSION =      1            2            3            4            5
==================================================================================================
 LOG-LIKELIHOOD                        -80084.828   -80014.835   -79991.273   -79976.945   -79995.617
 SAMPLE : - NUMBER OF OBSERVATIONS           8334         8334         8334         8334         8334
          - FIRST OBSERVATION                   1            1            1            1            1
          - LAST  OBSERVATION                8334         8334         8334         8334         8334
 NUMBER OF ESTIMATED PARAMETERS :
   - FIXED PART :
       . BETAS                                 10           10           10           10           10
       . BOX-COX                                0            1            2            2            2
       . ASSOCIATED DUMMIES                     0            0            0            0            0
   - HETEROSKEDASTICITY :
       . DELTAS                                 0            0            0            1            0
       . BOX-COX                                0            0            0            1            0
       . ASSOCIATED DUMMIES                     0            0            0            0            0
  ==================================================================================================
```

Appendix 4. Background Vacation Generation-Distribution Models (J. Last, July 1998)
BC-GAUHESEQ Family (LEVEL 1.4 algorithm)
Total Flow by all Modes; Vacation Trips (CAA/IPS 1991 and SES 1993-1994 data)

I. BETA	TYPE =	LEVEL-1	LEVEL-1	LEVEL-1	LEVEL-1	LEVEL-1
ELASTICITY E(y) (EP)	VARIANT =	vacE	vacE	vacE	vacE	vacE
PARTIAL DER. E(y) (EP)	VERSION =	1	2	3	4	5
(COND. T-STATISTIC)						
Origin characteristics						
Population	popxpsi3	.268185E+00	.267557E+00	.253895E+00	.249195E+00	.257382E+00
weighted by aPsi		.268	.268	.270	.260	.271
index based		.268799E-02	.259612E-02	.254093E-02	.236835E-02	.312355E-02
on delta values		(22.95)	(24.19)	(24.47)	(24.12)	(24.40)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
GDP per capita Orig	o_gdppc	.421698E+01	.439684E+01	.553640E+01	.757142E+01	.459273E+01
		.073	.076	.083	.118	.070
		.436881E+05	.440979E+05	.468570E+05	.646661E+05	.484124E+05
		(2.82)	(2.93)	(3.27)	(4.69)	(2.72)
Destination characteristics						
Population in Dest	d_pop	.130845E+00	.128807E+00	.117276E+00	.116978E+00	.125045E+00
		.131	.129	.125	.122	.132
		.116453E-02	.110981E-02	.104399E-02	.988402E-03	.134954E-02
		(8.31)	(8.17)	(8.02)	(7.98)	(8.42)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
Seaside at Dest	seaside	.140826E+00	.149710E+00	.173657E+00	.166952E+00	.167157E+00
but not in Orig	=======	.141	.150	.151	.152	.148
		.145897E+04	.150150E+04	.146974E+04	.142591E+04	.176202E+04
		(5.23)	(5.75)	(5.90)	(5.99)	(5.75)
Mountains in	mountain	.266888E+00	.269472E+00	.302186E+00	.309841E+00	.333824E+00
Dest but not in Orig	========	.267	.269	.263	.281	.295
		.276497E+04	.270266E+04	.255753E+04	.264630E+04	.351887E+04
		(6.08)	(5.50)	(5.50)	(6.17)	(6.20)
Destination/Origin ratio						
July	summer2	.107736E+01	.110877E+01	.123541E+01	.140984E+01	.117537E+01
temperature		1.077	1.109	1.105	1.296	1.075
Dest vs. Orig		.109273E+05	.108871E+05	.105256E+05	.119392E+05	.125502E+05
		(16.39)	(17.74)	(16.74)	(17.96)	(16.22)
		LAM	LAM	LAM	LAM	LAM
Impedances						
Cross-border journey	cross	-.356595E+00	-.334748E+00	-.369357E+00	-.311452E+00	-.345169E+00
	=====	-.357	-.335	-.321	-.283	-.305
		-.369435E+04	-.335734E+04	-.312604E+04	-.266005E+04	-.363846E+04
		(-5.70)	(-5.25)	(-5.18)	(-4.65)	(-4.84)
Common	lang2	.593108E+00	.558812E+00	.601275E+00	.568066E+00	.583605E+00
language in Orig and	=====	.593	.559	.523	.516	.516
Dest and		.614463E+04	.560458E+04	.508886E+04	.485175E+04	.615183E+04
Cross-border (Dummy)		(3.23)	(3.53)	(3.40)	(3.54)	(3.31)
Travel time (car)	time	-.230890E+00	-.386235E+02	-.279061E+02	-.764006E+01	
		-.231	-.172	-.201	-.265	
		-.393368E+01	-.284018E+01	-.322232E+01	-.409783E+01	
		(-8.71)	(-11.82)	(-12.95)	(-13.99)	
		LAM	LAM	LAM	LAM	
Utility index	util_int3					.316536E+00
[exp(util_int)]						.207
						.272647E+05
						(12.00)
						LAM
Database						
Dummy for	rec1912	.268868E+01	.263383E+01	.297105E+01	.281349E+01	.298565E+01
record # 1912	=======	2.689	2.634	2.585	2.555	2.640
		.278548E+05	.264159E+05	.251454E+05	.240295E+05	.314720E+05
		(2.85)	(.00)	(.00)	(.00)	(.00)

Appendix 4. (Continued)

```
  Screenline survey      ips_caa      -.187459E+01 -.186702E+01 -.212301E+01 -.204392E+01 -.199472E+01
                         =======            -1.875       -1.867       -1.847       -1.856       -1.763
                                      -.194208E+05 -.187252E+05 -.179680E+05 -.174568E+05 -.210265E+05
                                          (-31.60)     (-30.21)     (-30.70)     (-31.46)     (-28.77)

-----------------------------------------------------------------------------------------------------
  REGRESSION CONSTANT    CONSTANT      .597266E+01  .500113E+02  .420032E+02  .193384E+02  .561288E+01
                                                 -            -            -            -            -
                                                 -            -            -            -            -
                                           (19.86)      (12.98)      (14.69)      (18.33)      (21.07)
----------------------------
HETEROSKEDASTICITY STRUCTURE
----------------------------
  DELTA COEFFICIENTS
  ------------------
  Travel distance (car)  dist                                                 .107481E-17
                                                                                     .006
                                                                              .667496E-01
                                                                                  (10.07)
                                                                                      LAM

=====================================================================================================
II. PARAMETERS                  TYPE =  LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1
    and                      VARIANT =    vacE         vacE         vacE         vacE         vacE
   (COND. T-STATISTIC)       VERSION =     1            2            3            4            5
=====================================================================================================
----------------------------
HETEROSKEDASTICITY STRUCTURE
----------------------------
  BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]
  ------------------------------------------------------------------
  LAMBDA(Z)              dist                                                       5.514
                                                                                   [7.34]
                                                                                   [6.01]

------------------------------------------------------------------
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]
------------------------------------------------------------------
  LAMBDA(Y) - GROUP  1   LAM  1               .000         .000         .014         .010         .013
                                             FIXED        FIXED       [2.22]       [1.58]       [1.94]
                                                                   [-150.72]    [-155.21]    [-152.40]

  LAMBDA(X)              time                 .000        -.844        -.748        -.509
                                             FIXED      [-5.86]      [-5.01]      [-3.46]
                                                       [-12.80]     [-11.70]     [-10.24]

  LAMBDA(X)              summer2              .000         .000        1.314         .599        1.607
                                             FIXED        FIXED       [2.30]       [1.06]       [2.49]
                                                                       [.55]       [-.71]        [.94]

  LAMBDA(X)              util_int3                                                               .125
                                                                                                [1.81]
                                                                                              [-12.67]

  LAMBDA(X) - GROUP  1   LAM  1               .000         .000         .014         .010         .013
                                             FIXED        FIXED       [2.22]       [1.58]       [1.94]
                                                                   [-150.72]    [-155.21]    [-152.40]

=====================================================================================================
III.GENERAL STATISTICS          TYPE =  LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1      LEVEL-1
                             VARIANT =    vacE         vacE         vacE         vacE         vacE
                             VERSION =     1            2            3            4            5
=====================================================================================================
LOG-LIKELIHOOD                          -65407.101   -65383.550   -65374.566   -65313.214   -65390.296
SAMPLE : - NUMBER OF OBSERVATIONS             6540         6540         6540         6540         6540
         - FIRST OBSERVATION                     1            1            1            1            1
         - LAST  OBSERVATION                  6540         6540         6540         6540         6540
NUMBER OF ESTIMATED PARAMETERS :
  - FIXED PART :
      . BETAS                                   12           12           12           12           12
      . BOX-COX                                  0            1            3            3            3
      . ASSOCIATED DUMMIES                       0            0            0            0            0
  - HETEROSKEDASTICITY :
      . DELTAS                                   0            0            0            1            0
      . BOX-COX                                  0            0            0            1            0
      . ASSOCIATED DUMMIES                       0            0            0            0            0
=====================================================================================================
```

Appendix 5. O-D Flows : NUTS-3 Samples Used in Generation-Distribution Model Regressions

Business trips : NUTS-3 observations by source and NUTS-0 totals

0001-1054	SES Enquête transports 1993-1994 (domestic)
1055-1190	SES Enquête transports 1993-1994 (abroad)
1191-1761	IPS/CAA 1991 French residents (cross-channel)
1762-4220	IPS/CAA 1991 UK residents (cross-channel)
4221-5711	IPS/CAA 1991 other residents (cross-channel)

O/D	BE	CH	DE	DK	ES	FR	IR	IT	LU	NL	PT	UK	SUM
AT												1	1
BE												168	168
CH												20	20
DE												426	426
DK												14	14
ES												172	172
FR	18	15	45	1	11	1054	2	13		11	5	821	1996
IT												193	193
LU												23	23
NL												195	195
PT												42	42
SE												2	2
UK	194	1	467	2	300	829		347	42	201	76	0	2459
SUM	212	16	512	3	311	1883	2	360	42	212	81	2077	5711

Private trips : NUTS-3 observations by source and NUTS-0 totals

0001-2165	SES Enquête transports 1993-1994 (domestic)
2166-2377	SES Enquête transports 1993-1994 (abroad)
2378-3071	IPS/CAA 1991 French residents (cross-channel)
3072-6492	IPS/CAA 1991 UK residents (cross-channel)
6493-8334	IPS/CAA 1991 other residents (cross-channel)

O/D	AT	BE	CH	DE	ES	FR	IR	IT	NL	PT	UK	SUM
AT											4	4
BE											129	129
CH											8	8
DE											549	549
ES											329	329
FR		63	31	39	28	2165	1	18	14	3	972	3334
IT											312	312
NL											189	189
PT											59	59
UK	3	182	4	565	476	1235		624	226	106		3421
SUM	3	245	35	604	504	3400	1	642	240	109	2551	8334

Vacation trips : NUTS-3 observations by source and NUTS-0 totals

0001-2195	SES Enquête transports 1993-1994 (domestic)
2196-2517	SES Enquête transports 1993-1994 (abroad)
2518-3190	IPS/CAA 1991 French residents (cross-channel)
3191-5206	IPS/CAA 1991 UK residents (cross-channel)
5207-6540	IPS/CAA 1991 other residents (cross-channel)

O/D	AT	BE	CH	DE	ES	FR	IR	IT	NL	PT	UK	SUM
AT											40	40
BE											104	104
CH											43	43
DE											448	448
ES											120	120
FR		14	21	42	87	2195	3	69	7	38	914	3390
IT											136	136
NL											204	204
PT											39	39
UK	83	121	55	298	107	1019		133	189	11	0	2016
SUM	83	135	76	340	194	3214	3	202	196	49	2048	6540