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Introducing Spatial Competition through an Autoregressive Contiguous Distributed (AR-C-D) Process in Intercity Generation-Distribution Models within a Quasi-Direct Format (QDF)

by

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Abstract

We show how one type of autoregressive contiguous distributed process (AR-C-D) can be used to introduce spatial competition in transportation demand models that are generally consistent with Luce's IIA axiom because they make each $\underline{ij}$ flow depend only on $\underline{ij}$ transport and socioeconomic conditions. The particular process used is an R-Koyck process that, for each order of correlation used, adds to the usual autocorrelation parameter ρ a proximity parameter π that describes the relative weight of close and remote neighbours in an infinite distributed lag structure. The process selects relevant competing (or complementary) origin-destination pairs required to obtain spherically distributed residuals. The tests are carried out on representative intercity passenger flows for Canada and Germany using different contiguity matrices: natural and directed first and second order processes are studied with and without an associated distributed lag structure. The AR-C-D $(1, \infty, G(\pi))$ and AR-C-D $(2, \infty, G(\pi))$ parameters are naturally estimated jointly with heteroskedasticity and the functional form of the generation-distribution models. As these models are set within a quasi-direct format (QDF), the prerequired mode choice models make it possible to derive modal demand elasticities from the product of the total demand and mode choice models. As occurs every time systematic information is extracted from regression results to correct for model misspecification, the impact on both statistical and economic results, such as elasticities of demand, is important whether the usual multiplicative form or an optimal model form is used.

Keywords: General Autocorrelation; Spatial autocorrelation; distributed lag; R-Koyck lag; heteroskedasticity; Box-Cox transformations; natural order; directed order; AR-C-D process; intercity; total demand; mode choice; generation-distribution; Canada; Germany; elasticity; spatial competition.

Résumé

Nous montrons comment un type de processus autorégressif contigu et distribué (AR-C-D) peut être utilisé pour introduire la concurrence spatiale dans les modèles de demande de transport compatibles avec l'axiome IIA de Luce s'ils font dépendre chaque flux $\underline{ij}$ des conditions de transport ou économiques associées à cette paire $\underline{ij}$, à l'exclusion des autres paires. Le processus particulier R-Koyck utilisé ajoute au paramètre d'autocorrélation ρ d'un ordre donné un paramètre supplémentaire de proximité π qui décrit l'importance relative des voisins proches et éloignés appartenant à une structure de retards échelonnés de degré infini. Le processus choisit alors les paires origine-destination concurrentes (ou complémentaires) requises à l'obtention d'erreurs résiduelles à distribution sphérique. Nous utilisons pour les tests des flux interurbains de transport des voyageurs au Canada et en Allemagne et formulons diverses matrtices de contiguité, suivant des critères naturels et dirigés, pour des processus de premier et de second ordre, avec et sans leur ajouter une structure de retards échelonnés. Les paramètres supplémentaires impliqués par les processus AR-C-D $(1, \infty, G(\pi))$ et AR-C-D $(2, \infty, G(\pi))$ sont naturellement estimés conjointement avec les paramètres d'hétéroscédasticité et de forme fonctionnelle des modèles de génération-distribution utilisés. Comme par ailleurs ces modèles sont sis dans un format quasi-direct (QDF), les modèles de choix modal prérequis rendent possible le calcul d'élasticités de la demande par mode de transport dérivées du produit du modèle de demande totale et du modèle de choix modal. Nous constatons que l'impact sur les résultats statistiques et économiques – tels les élasticités de la demande – de l'exploitation de l'information systématique présente dans les résidus de régression est important et dans les modèles qui utilisent la forme multiplicative usuelle et dans ceux dont la forme optimale est estimée, comme il est fréquent lorsqu'une telle exploitation redresse en fait la formulation du modèle.

Mots-clés: Autocorrélation; Autocorrélation spatiale; retards échelonnés; retard R-Koyck; hétéroscédasticité; transformation Box-Cox; ordre naturel; ordre dirigé; processus AR-C-D; interurbain; demande totale; choix modal; génération-distribution; Canada; Allemagne; élasticité; concurrence spatiale.

Zusammenfassung

In dieser Arbeit wird ein Verfahren aus der Familie der autoregressiven, benachbarten und verteilten Prozesse (AR-C-D) dazu verwendet, räumliche Konkurrenz in Verkehrsnachfragemodelle zu integrieren. Diese Modelle entsprechen grundsätzlich Luces' IIA-Axiom, da jeder Strom von i nach j lediglich aus dem Transportangebot zwischen bzw. dem sozioökonomischen Werten von i und j resultiert. Das spezifische Verfahren ist ein R-Koyck-Verfahren, das für jede Form der Korrelation zu dem gewöhnlichen Autokorrelationsparameter ρ einen Lagegunstparameter π einführt, der den Einfluβ naher und ferner Nachbarn im Raum beschreibt. Das Verfahren selektiert relevante konkurrierende (oder komplementäre) Quelle-Ziel-Beziehungen, um sphärisch verteilte Residuen zu erhalten. Getestet werden repräsentative Reisendenströme für Kanada und Deutschland mit unterschiedlichen Distanzmatrizen: natürliche und reglementierte Verfahren 1. und 2. Ordnung werden mit und ohne die zugehörigen spärisch verteilten Residuenstrukturen untersucht. Die Schätzung der AR-C-D $(1, \infty, G(\pi))$- und AR-C-D $(2, \infty, G(\pi))$-Parameter erfolgt unbeschränkt und gemeinsam mit der Heteroskedastizität sowie der funktionalen Form der Erzeugung-Verteilungsmodelle. Da diese Modelle in einem quasi-direkten Format (QDF) vorliegen machen es die vordefinierten Verkehrsträgerwahl modelle möglich verkehrsträgerspezifische Nachfragelastizitäten aus dem Produkt von Gesamtnachfrage und verkehrsträgerspezifischen Anteilen herzuleiten. Wie immer werden aus den Regressionsergebnissen systematische Informationen herausgefiltert, um Modellfehler zu korrigieren. Dabei ist es wichtig, ob die statistischen und ökonomischen Ergebnisse, wie z.B. die Nachfrageelastizitäten, auf einer multiplikativen oder optimalen Modellform basieren.

Stichworte: Generelle Autokorrelation; räumliche Autokorrelation; Heteroskedastizität; Box-Cox-Transformation; Verteilungsdelta; R-Koyck-Delta; natürliche Ordnung; reglementierte Ordnung; AR-C-D-Verfahren; Intercity; Gesamtnachfrage; Modalwahl; Erzeugung-Verteilung; Kanada; Bundesrepublik Deutschland; Elastizität; räumliche Konkurrenz; Residuenstruktur.

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1. Generation-distribution models: structure[1]

The basic intent of generation-distribution models is to explain T_{ij}, the travel flow from $\underline{i}$ to $\underline{j}$ in terms of two classes of functions. Some, the A_{ij_a}, refer to activity levels at the origin or destination and others, the U_{ij_m}, represent the utility of the $\underline{m}$ travel modes:

$$T_{ij} = g^d(\{A_{ij_a}\}, \{U_{ij_m}\}) \ , \qquad a = 1, ..., A \ ; \ \ m = 1, ..., M \ . \tag{1}$$

A frequent specification, for the first type, is the geometric mean of values of the activity at the origin and destination, for instance

$$A_{ij_a} \equiv \left[S_{i_a}^{1/2} \ S_{j_a}^{1/2} \right] \ , \tag{2}$$

where S_a is a socioeconomic variable, such as population, or income.

A frequent specification for the second type is

$$U_{ij_m} = e^{V_{ijm}} \tag{3}$$

where V_{ij_m} is primarily a function of network variables N_{ij_n}, such as cost, time, distance or "impedance", but also frequently of socioeconomic variables S_a as well.

Because of the complexity of the problem, it is practical to aggregate the modal utilities and define an index

$$U_{ij} = \sum_m e^{V_{ijm}} \tag{4}$$

which in effect constitutes the denominator of a logit model, and makes it possible to write the usual form

$$T_{ij} = \beta_o \, A_{ij_1}^{\beta_1} \ ... \ A_{ij_A}^{\beta_A} \, U_{ij}^{\beta_U} \, u_{ij} \ \ , \tag{5}$$

where u_{ij} is an error term. Note that the coefficient β_U is expected to be positive, as an increased utility of travel modes should lead to higher flows, and that the signs of the activity

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coefficients will depend on the exact specifications used for each activity variable: in a simple model using, say Population, the coefficient of that term should be positive; however, if variables like the proportion of jobs belonging to a particular sector are used, their coefficients may well be negative.

Note that formulation (5) does not use double constraints (to insure that trips from any origin – or to any destination – add up to an observed total) because such constraints are inconsistent in models that explain the number of trips[2]. In fact, even in models in which one is trying to explain the distribution of trips (as opposed to their level) originating or destinating in a zone, O_i and D_j, as in

$$T_{ij} = k\ O_i\ D_j\ U_{ij}^{\gamma} u_{ij} \quad , \tag{6}$$

the imposition of constraints may bias the estimation of the remaining γ parameter because of the restrictions implied for the residual error. For both reasons, we shall proceed with (5) as a representative generation-distribution format.

In any case, models of either format have three difficulties that our approach will address.

2. Functional form and stochastic specification: economic, mathematical and statistical issues

To discuss the three difficulties that arise, it is convenient to rewrite (5) as a regression problem, using the subscript $\underline{t}$ instead of $\underline{ij}$ to denote origin-destination pairs, and X_k to denote either the activity variables or the modal utility index

$$y_t^{(\lambda_y)} = \beta_0 + \sum_k \beta_k\ X_{k_t}^{(\lambda_k)} + u_t \tag{7-A}$$

$$u_t \quad = [f(Z_t)]^{1/2}\ v_t \tag{7-B}$$

$$v_t \quad = \sum_\ell \rho_\ell \left[\sum_n \tilde{r}_{\ell,t_n} v_n \right] + w_t \tag{7-C}$$

where the (λ) denote the Box-Cox Transformation (BCT) and the meaning of the stochastic specification will be clarified, as each difficulty is stated and resolved.

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[2] Balancing factors, like dummy variables D_i and D_j, would be exactly colinear with variables such as S_i and S_j if those were used in a regression to explain T_{ij}. Balancing factors assume that trip ends T_i and T_j are observed without error. They in fact make the adjustment of non ij flows that follow a change in U_{ij} unpredictable. Moreover, all solutions must have the unlikely property that the total cost over all flows is constant.

2.1 Functional form

The BCT has the advantage of including the linear $(\lambda = 1)$ and logarithmic $(\lambda = 0)$ cases as special nested values. For instance, if $\lambda_y = \lambda_k = 0$ in (7-A), the multiplicative form (5) is obtained. As generation-distribution models are normally specified to be multiplicative, the question naturally arises whether one can improve the fit by allowing the λ parameters to differ from 0. Indeed Gaudry and Wills (1978) have shown that one could improve the fit obtained for an intercity model specified with data for Canada in 1972. Since, much work by many authors has shown that the BCT transformation is a powerful tool for any problem where an empirically good fit is of interest.

An important subproblem in generation-distribution models is whether the modal utility index U enters multiplicatively or not. The reason for this is that the natural logarithm of the denominator of the logit model, $\ell n[U] = \ell n\left[\sum_m e^{V_m}\right]$, is known to represent the expected value of the maximum utility available to the consumer over all travel modes or alternatives, and is often called the inclusive value or price of the modes (Williams, 1977). The Gaudry and Wills paper mentioned above could not really reject the logarithmic form $(\lambda = 0)$ for this term, despite the fact that it could easily reject it for the other terms of the generation distribution model.

The form issue is clearly important as an economic behaviour problem. The computational burden of the additional parameters has however slowed the adoption of the BCT as a simple extension of regression analysis. In addition, it has recently been pointed out (Spitzer, 1984) that the BCT poses a special statistical problem if one is interested in obtaining unconditional *t*-statistics for the β_k coefficients: these *t*-values then depend on units of measurement of the X_k, a very surprising result. To get around this problem, it is convenient to compute conditional *t*-statistics for the β_k parameters, namely *t*-values that depend on the estimated BCT values considered as given. This means that the resulting *t*-values overestimate somehow the "true" values that would have been obtained if the fact that the BCT parameters are estimated had been taken into account.

2.2 The size distribution of the error term

It is well known that proper statistical inference about the various model parameters requires that the variance of the error term u_t be constant. In many regression analyses, this problem is not adequately examined. It is of some urgency to face it here because it is clear that the application of transformations, in particular to the dependent variable, will change the error variance. For instance, using the logarithmic form $(\lambda = 0)$ of the dependent variable should decrease high values of that variable proportionately more than small values, and concur both to reducing the size of the error variance, and to making that variance more constant. This way of obtaining a hopefully constant variance (homoskedastic) error term is known to practioners who try out different forms and often find that they get "better" *t*-statistics with a logarithmic transformation than if they leave the dependent variable untransformed. However, it should not be thought that the relationship is simple or automatic because, as illustrated in Dagenais, Gaudry and Liem (1987), some transformations can induce the error term variance to become non constant (heteroskedastic). This is really not surprising and leads to the double-barrel strategy of defining specific tools to maintain homoskedasticity as one simultaneously estimates the functional form. The explicit form of $f(Z_t)$ in (7-B), where the Z stands for a set of potential variables Z_m, is

$$f(Z_t) = \exp\left[\sum_m \delta_m Z_{m_t}^{(\lambda_{zm})}\right] \quad , \tag{8}$$

which insures both a positive and constant variance for v_t. Note that classical heteroskedasticity Z_t^2 is obtained by setting all λ_{zm} and all δ_m but one equal to 0.

In addition to its statistical importance, obtaining a constant variance by finding an appropriate form for $f(Z_t)$ can be seen as extracting functional information from the error term u_t. If no such model can be specified (i.e. if $f(Z_t) = 1$), and $u_t = v_t$, we can say that no "contemporaneous" variable[3], that is no Z_{m_t} variable was found to determine the "current" value of the error u_t. Here the symbol Z_m denotes a variable used explicitly in (8) and is not meant to exclude its simultaneous use in (7-A) and (7-B) under certain conditions – for instance that (7-B) contain more than one variable. One of the Z_m variables may simply also be an X_k variable that obtains two roles: explain the level of y_t and the variance of u_t as well!

In economic terms, this should not be surprising. Consider for a moment our matrix of travel flows T_{ij}. Because flows vary considerably in size, it would be extremely lucky for a model to yield residual errors of roughly the same size (variance).

2.3 The problem of spatial competition or structure of the O-D matrix

The current specification of generation-distribution models indicated in (5) relates the origin-destination flow for each ij pair solely to factors associated with the origin $\underline{i}$, the destination $\underline{j}$ or both, as for the utility index U_{ij}. The reason for the exclusion of variables associated with other pairs, for instance U_{ik}, or U_{hj}, or U_{hk} for that matter, is multicollinearity. This means that, because of the difficulty of introducing additional terms to represent the attractiveness or access cost of competing destinations (or origins), the flow from $\underline{i}$ to $\underline{j}$ does not depend on the

[3] Leads or lags of Z_{m_t} can also be used in time series models.

characteristics of these other destinations (or origins), or on the disutility of travel between such locations and those currently considered, namely $\underline{i}$ and $\underline{j}$.

Stated more formally, the absence of competition implies that the models are consistent with Luce's IIA (independence from irrelevant alternatives) axiom because the ratio of two "choices" depends only upon the characteristics of these two. In our case (and neglecting the activity variables to simplify):

$$\frac{T_{ij}}{T_{ik}} = \frac{U_{ij}^{\beta_U}}{U_{ik}^{\beta_U}} \quad . \tag{9}$$

As a consequence, the modification of modal utility for another pair, say U_{iq}, has no effect on the above ratio. So the introduction of a High Speed Rail line will not change the structure of the matrix: it will only affect flows of pairs for which the own U_{ij} has been modified.

Although understandable as a simplification, the inclusion of only <u>own</u> utilities would be expected to bias the estimated price or travel time elasticities upwards because no part of any given flow can be explained by substitution effects, or for that matter by complementarity effects – however complementary zones are likely to be rare and might require specific treatment. We may conjecture that this structural feature of generation-distribution models may be responsible for some of the high values of elasticities often found in cross-sectional models – they tend to be higher than those derived from time-series models.

As this problem is very important, we shall presently discuss it at length, assuming for the moment that locations are substitutes.

3. Approaches to the problem of spatial competition

In an intuitive sense, the absence of <u>cross</u> terms in (5) should be remedied by their inclusion, if the original problem that led to the simplification – multicollinearity – can be minimized or the situation made tolerable. We have examined two ways of including <u>non</u> ij terms in the explanation of ij flows: a direct one, due to Wills (1986), and an indirect one formulated by Blum, Bolduc and Gaudry (1996). Although both papers contained illustrations, none included a full-blown example of the proposed approach. After due analysis, we opted for the latter, for reasons that will be clear shortly.

3.1 <u>Wills' direct approach</u>

Wills (1986) introduces the <u>cross</u> terms (non ij) by writing a generalization of the modal utility index (4) and calling it "proportionality factor". It is defined as

$$\Pi_{ij} = \left[\left(\sum_{k}^{j} U_{ik} \right)^{(\lambda_0,\tau)} - \left(\sum_{k}^{j-1} U_{ik} \right)^{(\lambda_0,\tau)} \right] , \tag{10-A}$$

$$U_{ik} = \exp \left[(1 - \Psi_0) \sum_{a} \alpha_a \, S_{ika}^{(\lambda_a)} + \sum_{n} \gamma_n N_{ikn}^{(\lambda_n)} \right] , \tag{10-B}$$

where, Ψ_0 denotes a blending parameter which insures that, when it is set to 1, the gravity model is obtained, (λ_a), (λ_n) denote Box-Cox transformations (BCT), (λ_0, τ) denotes a convex combination of terms, the first of which is transformed by a direct BCT and the second by an inverse BCT, as in

$$y^{(\lambda_0,\tau)} = \tau \; y^{(\lambda_0)} + (1 - \tau) \; y^{\left(\lambda_0^{-1}\right)} \; , 0 \leq \tau \leq 1. \tag{10-C}$$

The generality of this specification comes from the fact that (10–A) includes a difference in the sum of two functions – this means that all destinations intervene in determining the ij flow – and from the fact that the nonlinear transformation in (10–A) gives a differential impact to the same terms of the cumulative sum (as the sums differ only by one term). Although this generality makes it possible to include as nested special cases both the usual generation-distribution format and the intervening opportunities format, a major problem of interpretation of (10–A) arises because it is not clear what happens over and above the fact that all modal utilities affect the generalized modal utility (now called proportionality factor) of the ij pair. Moreover, the combination of (10-A) and (10-B) is extremely nonlinear: the resulting complexity motivates Wills to use for his illustration an extremely simple example and specify a multiplicative form for (7-A) and (10-B) obtained by setting the BCT equal to 0 a priori. Finally, the procedure requires an ordering of destinations that both depends on the criterion used and complicates the programming and interpretation. For these reasons, we decided to resort to a more straightforward and computationally tractable approach.

3.2 The Blum-Bolduc-Gaudry indirect approach

A. The notion of spatial correlation

The basic problem of the usual model specification (5) is that it cannot capture all relevant factors related to the geographic structure. For instance, if two contiguous destinations 2 and 7 in Figure 2 have shopping and tourism facilities, these destinations may be close substitutes for shoppers and tourists from origin 1, so that the flows should be explained as

$$T_{12} = f(\ A_{12},\ A_{17},\ U_{12}, U_{17}) + v_{12}$$

(FULL)

$$T_{17} = f(\ A_{12},\ A_{17},\ U_{12}, U_{17}) + v_{17}$$

rather than as

$$T_{12} = f(\ A_{12},\ -\ ,\ U_{12},\ \ -\ \ ) + v_{12}$$

. (SPARSE)

$$T_{17} = f(\ -\ ,\ A_{17},\ \ -\ \ , U_{17}) + v_{17}$$

It would not be surprising if the omitted[4] variables in the formulation (SPARSE) caused its error terms to be positively correlated. In addition, the aggregation error arising from zonal boundary definitions, trip length cut off criteria used in defining the O-D matrix, and other sampling difficulties lead us to suspect that the error terms of contiguous zones will not be independent (could be positively or negatively correlated). In fact the burden of proof should no doubt fall on anyone claiming that the model is perfect enough to have spatially independent error terms!

But then, what would the knowledge of this correlation in the formulation (SPARSE) do for us? If we knew that

$$v_{12} = \rho\ v_{17} + w_{12} \quad , \tag{R-IMPACT}$$

we could combine (SPARSE) and (R-IMPACT) in this way:

$$T_{12} = f(A_{12}, U_{12}) + \rho\ (T_{17} - f(A_{17}, U_{17})) + w_{12} \quad , \tag{CHOSEN}$$

where it would be clear that (CHOSEN), by including everything that is missing in (SPARSE), is an approximation of (FULL) and that the autocorrelation parameter ρ weighs the role of competing contiguous flows in explaining the "current" flow. The more formal statement of (R-IMPACT) is, for any O-D flow t,

$$v_t = \rho \sum_{n=1}^{N} r_{tn} v_n + w_t \quad , \qquad (t,\ n = 1, ..., N) \ , \tag{11}$$

where r_{tn} equals 1 if pairs t and n are considered contiguous and 0 otherwise. In this way, the factors that explain contiguous flows (as well as the values of these flows) all contribute to the explanation of the current flow.

[4] In that sense, these become common factors that cause the IIA property by their absence across alternatives.

The formulation of such groups of contiguous observations, of “near” neighbours, implies the formulation of a matrix R of 1 and 0 values stating the connection between any observation t and any other observation n.

Indeed, another way of writing (11) and extending it to a second set of “near neighbours” is simply

$$v_t = \sum_{\ell} \rho_\ell \left[\sum_{n}^{N} r_{\ell,tn} v_n \right] + w_t \qquad (\ell = 1, 2) \tag{12}$$

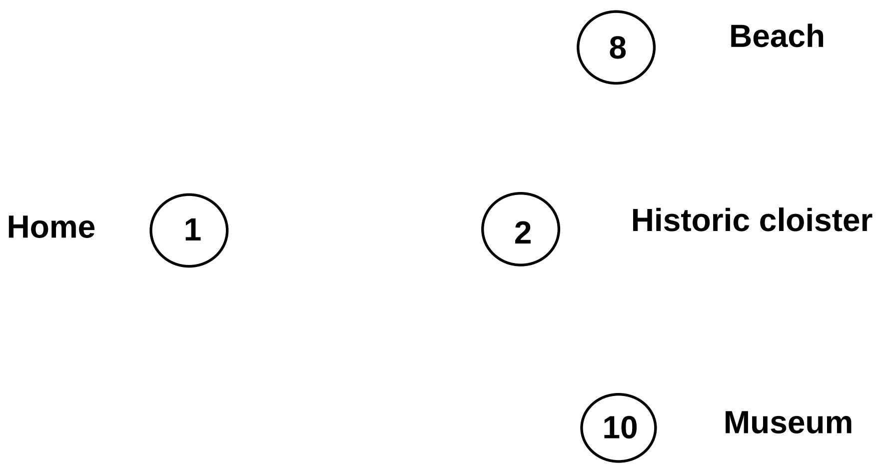
which has the same form as (7-C) except for a tilde (˜). This notational difference is required for two reasons. First, in practice, regression models with spatially autoregressive residual structures such as (11) or (12) will not yield a convex likelihood function unless the matrix R is subjected to either row (or column) normalization, a procedure which involves dividing each line (or column) by its sum. This normalization, expressed by using a bar (-) on the matrix, as in $\bar{R}$, has long been known as a practical device (e.g. Ord, 1975) but does not appear to have been proven until Bolduc (1985, 1987) examined the resulting statistical properties of the estimators. Note that this proof opens the door to the specification of arbitrary $\bar{R}$ matrices, that is to matrices where the criterion of “near neighbourliness” is arbitrary and need not depend on a natural order (“NO”, given by time or space) but may depend on any order directed (“DO”) by the analyst with a view to capturing a mispecification of the original problem (Gaudry and Blum, 1988).

But the fact that factors explaining contiguous observations are made to intervene to explain current observations helps us to understand something else about the role of autocorrelation. To see this, rewrite (CHOSEN) a little more formally in a linear format as

$$T_{ij} = \rho \, T_{ik} + \beta(U_{ij} - \rho \, U_{ik}) + w_{ij} \tag{13}$$

where the A terms have been dropped for simplicity. Note that, if $\rho > 0$, the impact of U_{ik} on T_{ij} will be in the opposite direction from that of U_{ij}, as occurs when goods are substitutes, and an improvement in the modal utility U_{ik} will reduce the T_{ij} flow. Similarly, a $\rho < 0$ will have the opposite effect, as occurs when goods are complements. This makes it conceivable to select R matrices in such a way as to reflect anticipated patterns of substitution or complementarity. In Figure 1, for instance,

Figure 1. Origin and potentially complementary destinations



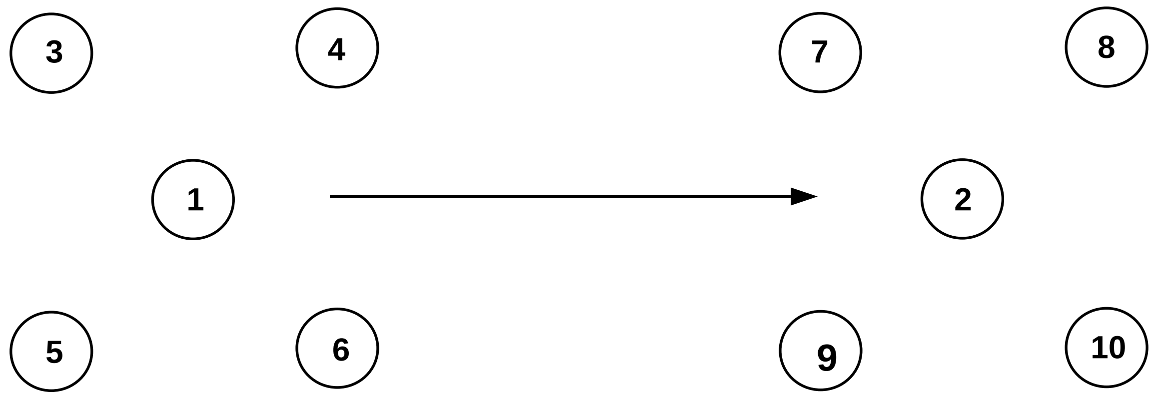
assume that $\boxed{1}$ is home and that $\boxed{2}$ is an historic cloister, $\boxed{8}$ a beach and $\boxed{10}$ a museum. When destinations are complements, as one would suspect if the data contain a lot of "tours" (i.e. the O-D matrix is far from symmetric for the flows among these zones), it is possible that selecting a naturally ordered (NO) residual impact criterion (RIC) such as

$$\text{Let } r_{tn} \begin{cases} = 1 & \text{if the flow is very asymmetric or part of a frequently observed tour;} \\ = 0 & \text{otherwise.} \end{cases} \qquad \text{(NORIC-1)}$$

to define the impact matrix R would lead to the estimation of a negative ρ. In that case higher transport prices for flows connecting an origin to complementary destinations would decrease trips to all such destinations: fewer trips to the beach, the cloister and the museum; conversely lower transport costs would raise flows to all complementary destinations. One may therefore conceive simultaneously of $\rho_1 > 0$ (substitutes) and $\rho_2 < 0$ (complements) within the same dataset, as long as one defined two appropriate RIC, one yielding R_1 and the other R_2.

Here our first concern will be to treat pattern of trips where substitution is expected to dominate, using symmetric matrices in order to reduce the size of the R matrices. It is convenient to consider the representation shown in Figure 2, where the

Figure 2. Origin and destination with contiguous points



arrow designates the symmetric flow from origin $\boxed{1}$ to destination $\boxed{2}$ and each of the zones involved in defining the "current" flow is surrounded by a set of contiguous near neighbours. A first RIC could assume that the error term v_{12} is correlated with all flows sharing the same destination, namely (3,2), (4,2), (5,2) and (6,2):

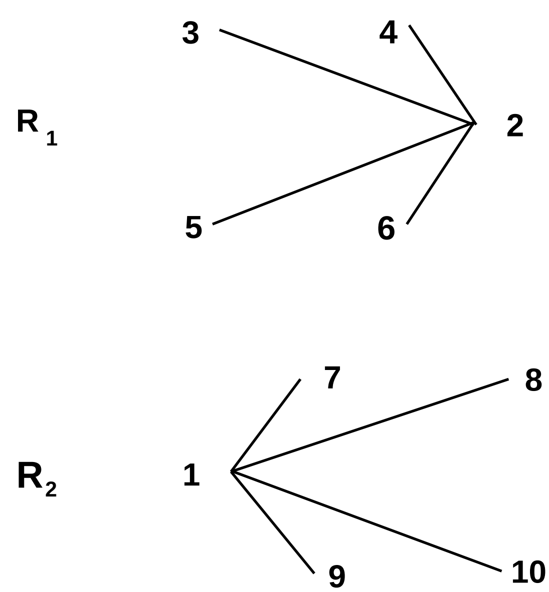
$$\left| \text{Let } r_{1,tn} \quad \begin{cases} = 1 & \text{if the flow has the same destination and is a near neighbour of the origin;} \\ = 0 & \text{otherwise.} \end{cases} \right| . \qquad \text{(NORIC-O)}$$

Similarly, one could define

$$\left| \text{Let } r_{2,tn} \quad \begin{cases} = 1 & \text{if the flow has the same origin and is a near neighbour of the destination;} \\ = 0 & \text{otherwise.} \end{cases} \right| . \qquad \text{(NORIC-D)}$$

Both (NORIC-O) and (NORIC-D) are illustrated in Figure 3.

Figure 3. Origin and destination contiguity



Naturally, another criterion could be the union of NORIC-O and NORIC-D

$$\text{Let } r_{tn} \begin{cases} = 1 & \text{if either the flow has the same destination and is a near neighbour of the origin, or conversely;} \\ = 0 & \text{otherwise.} \end{cases} \quad , \text{ (NORIC-OD)}$$

where the r_{tn} are elements of the N×N RIC matrix that defines "near neighbourliness", or contiguity:

$$R = \begin{bmatrix} 0 \,.\,.\,.\,.\, r_{1n} .\,.\,.\,.\, 1 \,.\,.\,.\,.\, 0 \\ : \qquad\qquad\qquad\qquad : \\ r_{t1} ......0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \\ : \qquad\qquad\qquad\qquad : \\ 0 \quad 0 \quad 1 \quad 1 \,.\,.\,.\,.\,.\,.\,.\,.\,.\, 0 \end{bmatrix} . \qquad (14)$$

B. Near and far neighbours: degrees of contiguity

Are there degrees of neighbourliness? Prompted by Gaudry who thought that one could find a spatial analog to the distributed lags of time series, Blum (1987) formulated a first process that was later redefined and extended (Blum, Bolduc and Gaudry, 1996). That paper shows that, if one considers

i) a sequence of contiguity matrices where the power c associated with the $\bar{R}^c$ matrix defines a degree of neighbourliness ($\bar{R}^2$ defines neighbours of neighbours, and so on)

$$\bar{R},\ \bar{R}^2,\ ...,\ \bar{R}^c, \tag{14-A}$$

ii) a normalized series of non-negative weights declining geometrically

$$\sum_{c=1}^{\infty} a^{c-1}(1-a) = 1, \qquad c = 1, ..., \infty \text{ and } a\ \epsilon(0,1)\ , \tag{14-B}$$

then the process of weighted contiguity matrices of order c

$$v = \rho\,(1-a) \sum_{c=1}^{\infty}\ a^{c-1}\ \bar{R}^c\ v + w \tag{14-C}$$

converges to

$$\begin{aligned} v =& \rho\,(1-a)\ \left[I - a\ \bar{R}\right]^{-1}\ \bar{R}\ \ v + w \\ =& \rho\ \pi\ \left[I - (1-\pi)\bar{R}\right]^{-1}\ \bar{R}\ \ v + w \\ =& \rho\ \tilde{R}\ \ v + w\ \ , \end{aligned} \tag{14-D}$$

where clearly $(1-a) = \pi$ and $\tilde{R} = \pi\ \left[I - (1-\pi)\ \bar{R}\right]^{-1}\ \bar{R}$.

This R-Koyck process[5] simply weighs power transformations of the contiguity matrix $\bar{R}$ by a series of geometrically declining weights. The powers of $\bar{R}$ allow backward and forward linkages arising through the first degree contiguity matrix, and the normalized sequence of weights allows a single proximity parameter π to describe the relative importance of near and distant neighbours.

If $\pi = 1$, we obtain the previous

$$v = \rho\ \bar{R}\,v + w \quad , \tag{15}$$

and if $\pi \rightarrow \epsilon$ (close to zero), we obtain

$$v = \rho\ \bar{\bar{R}}\,v + w \quad , \tag{16}$$

[5] The classical Koyck-lag of time series is in effect a special case.

where $\bar{\bar{R}}$ is a matrix with identical rows. In the latter case, proximity is low, i.e. the relative weight of distant neighbours is high; in the former, proximity is high, i.e. the relative weight of distant neighbours is low (is in fact zero). In effect, the relative importance of near and far neighbours has been endogenized.

The actual form of (7-C) if two orders of autocorrelation are present can perhaps best be seen in vector format

$$v = \rho_1 \, \tilde{R}_1 \, v + \rho_2 \, \tilde{R}_2 \, v + w \tag{17}$$

where $\tilde{R}_\ell = \pi_\ell \big[I - (1 - \pi_\ell) \, \bar{R}_\ell \big]^{-1} \bar{R}_\ell$ and we note the fact that the tilde (˜) denotes the autoregressive contiguous distributed AR-C-D $(\ell, \infty, G(\pi_\ell))$ process[6], that is a process autoregressive of order ℓ, contiguous of degree infinity and distributed with proximity π.

C. The likelihood function

Under the assumption that the error term w is a normally distributed white noise of variance σ_w^2, the log likelihood of y, for the system (7-A)-(7-B)-(7-C) written in matrix notation is

$$\ell n(L) = -\frac{N}{2} \ell n \big(2\pi \sigma_w^2 \big) - \frac{1}{2\sigma_w^2} \, w' w + \ell n |\det P| - \frac{1}{2} \sum_t \ell n [f(Z_t)] + (\lambda_y - 1) \sum_t \ell n \; y_t \, , \quad \text{(18-A)}$$

where

$$
\begin{aligned}
w \quad &= P \Big(H^{-1} \, y^{(\lambda_y)} - H^{-1} \, X^{(\lambda_x)} \, \beta \Big) \quad &, \quad (18 - B) \\
P \quad &= I - \rho_1 \, \tilde{R}_1 - \rho_2 \, \tilde{R}_2 \quad &, \quad (18 - C) \\
f(Z_t) \; &= \exp \left[\sum_m \delta_m Z_{mt}^{(\lambda_{zm})} \right] \quad &, \quad (18 - D) \\
H \quad &= \begin{bmatrix} \sqrt{f(Z_1)} & & 0 \\ & \ddots & \\ 0 & & \sqrt{f(Z_N)} \end{bmatrix} \quad &, \quad (18 - E)
\end{aligned}
\tag{18}
$$

and we require

$$-1 < \rho_\ell < 1 \quad \text{and} \quad 0 < \pi_\ell \leq 1 \quad . \quad (18 - \text{F})$$

The careful reader will note that the logarithm of the Jacobian of the transformation from w to v is expressed as $\ell n \, |\det \; P|$, rather than the equivalent expression $\sum_{t=1}^{N} \ell n [1 - \rho \, \gamma_t]$, where

6 The Blum et al. (1996) paper also describes processes with different degrees of contiguity and distributed according to other rules.

γ_t denotes the eigenvalues of the RIC impact matrix $\tilde{R}$, often used (e.g. Ord, 1975; Blum, Bolduc and Gaudry, 1996) because this simplification requires that the RIC impact matrix be susceptible to diagonalization – often an unrealistic assumption with asymmetric matrices such as $\widetilde{R}$ – and is in any case applicable only if a single order of autocorrelation is considered. This means that our procedure cannot avoid inverting $\widetilde{R}_\ell$ throughout the maximization procedure (Liem and Gaudry, 1994a) defined in terms of the parameters $\beta_1 ... \beta_k$, λ_y, $\lambda_{x_1}, ..., \lambda_{x_k}$ for (7-A), $\delta_1, ..., \delta_m$ and $\lambda_{z1}, ..., \lambda_{zm}$ for (7-B) and ρ_1, π_1, ρ_2, π_2 and σ_w^2 for (7-C). Although our procedure behaves well from the numerical point of view, we have not studied its statistical properties as an estimator: we assume that the properties holding in the case of multiple-order serial correlation (Dagenais, Gaudry and Liem, 1987) and first order spatial processes (Bolduc, 1985, 1987) hold when 2 orders of spatial autocorrelation are considered in (18).

4. Application to Germany, 1985

We will now describe the various choices that were made to test our approach with data for Germany 1985. Our purpose was to specify components of sufficient realism to demonstrate the usefulness of the approach. Major decisions had to be taken on the selection of a sample of Origin-Destination flows, on the specification of a mode choice model to determine modal utilities, and on a generation-distribution model. We examine these 3 major components in turn.

4.1 Flow selection and residual impact criteria

A. Symmetry of the O-D matrix, and other flow selection criteria

In view of the lack of numerical experience with the new algorithm, and in particular of the necessity of defining an R matrix of a size that does not require too much computer time for the repeated inversion of P in (18), we decided firstly to construct symmetric flows by taking the arithmetic average of the directional flows T_{ij}^* available in the existing 282 $\times$ 282 asymmetric matrix of total travel flows among zones of Germany in 1985:

$$T_{ij} = \frac{T_{ij}^* + T_{ji}^*}{2} \quad . \tag{19}$$

Although this introduced some observational errors and biased the demonstration of the use of the new technique by emphasizing the competitiveness of destinations (in spite of the partial offset provided by the autoregressive process, as will be seen shortly), it was thought preferable to doubling the size of the matrix because, with 286 observations drawn from the O-D matrix, the resulting R matrices of size 286 $\times$ 286 implied that up to 5 or 6 hours could be required on a SPARC 10 station for some of the experiments reported below.

Secondly, as both the mode choice model used to obtain modal utilities and the generation-distribution model proper were designed to account for strictly positive flows[7], we decided after some experimentation to accept the sample of 286 distinct pairs obtained by applying the following selection criteria and motivation

i) at least 10 000 trips: in order to reduce the sampling errors that tend to increase (relatively) as the total flow decreases;

ii) at least 1 % of the market belonged to each of the 3 modes (air, train, car): in order to reduce sampling errors associated with small flows and avoid some estimation difficulties that may arise in share models if too many minuscule shares are considered. Requiring a minimum market share of 5 % for each of the modes would have decreased the sample to 258 pairs;

iii) at least 100 kilometers between the origin and destination: in order to remove a reasonable proportion of flows that are more urban than intercity in nature.

B. Chosen autoregressive contiguous distributed processes

We decided to specify three R matrices, using (NORIC-O), (NORIC-D), and (NORIC-OD) and simultaneously defining contiguity as a neighbour situated between 100 and 180 km by car from the zone centroid in question. Using a belt of 100-180 kilometers to define neighbours mitigates the inconvenience of symmetry resulting from aggregation rule (19) because zones that are within that belt are less likely to be complements than nearer zones. So the expected sign of any of the autoregressive parameters $\rho_1, \ \rho_2, \text{ or } \ \rho$, associated with the three matrices is therefore positive. The application of these three rules yielded different numbers of 0 lines in the R matrices as can be seen in Table 1. Lines containing only zeroes are accepted in the algorithm and do not pose inversion problems because the identity matrix I in (18-C) compensates.

Two other comments should be made about the significance of the R matrices. The first one is to note that, when the origin and destination considered are far apart, the belts do not overlap, as is illustrated in Figure 4, where it is assumed, as in Figure 3, that there are four neighbours both at the origin and destination. However, when belts overlap – the greatest overlap does not occur when the distance between the centroïds of interest is the smallest (100 km) –, as in Figure 5, some points that are in the intersection of the belts can be in both R_1 and R_2, as is the case for 2 of the points shown.

[7] The SHARE S-1/S-5 procedure used (Gaudry, Dagenais, Laferrière, Liem, 1993) allows the market share model to be estimated under certain conditions even if some modes are not available for some pairs, but we decided not to use this option because of the restrictive assumptions needed for such cases.

Figure 4. R-Matrix elements without overlapping rings

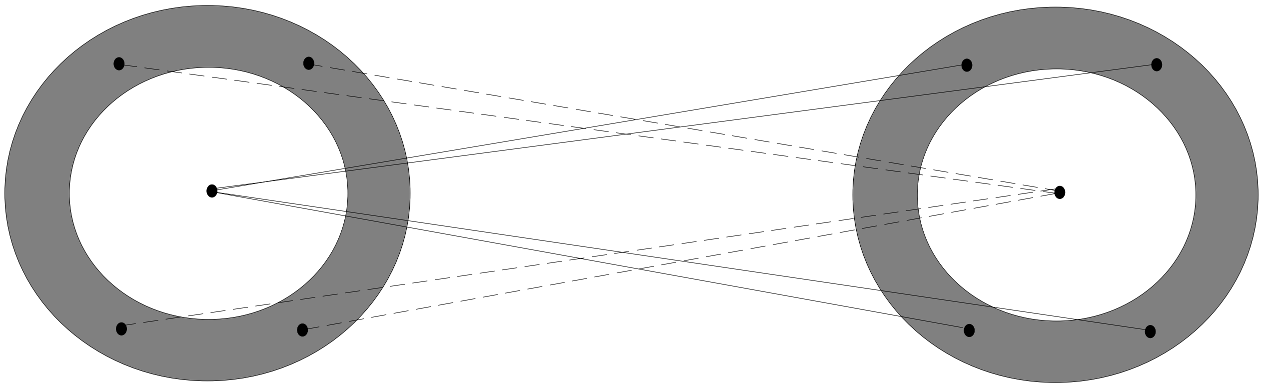


Figure 5 R-Matrix elements with partially overlapping rings

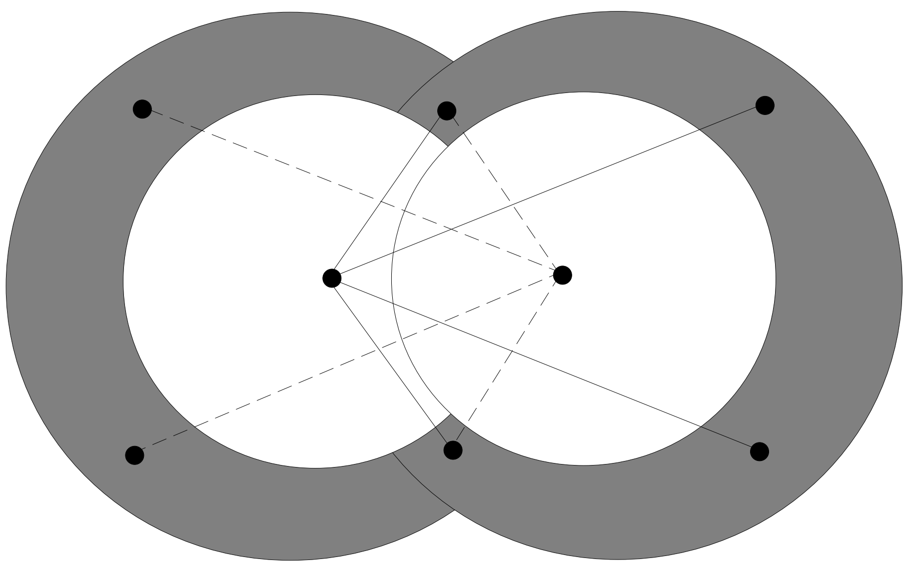
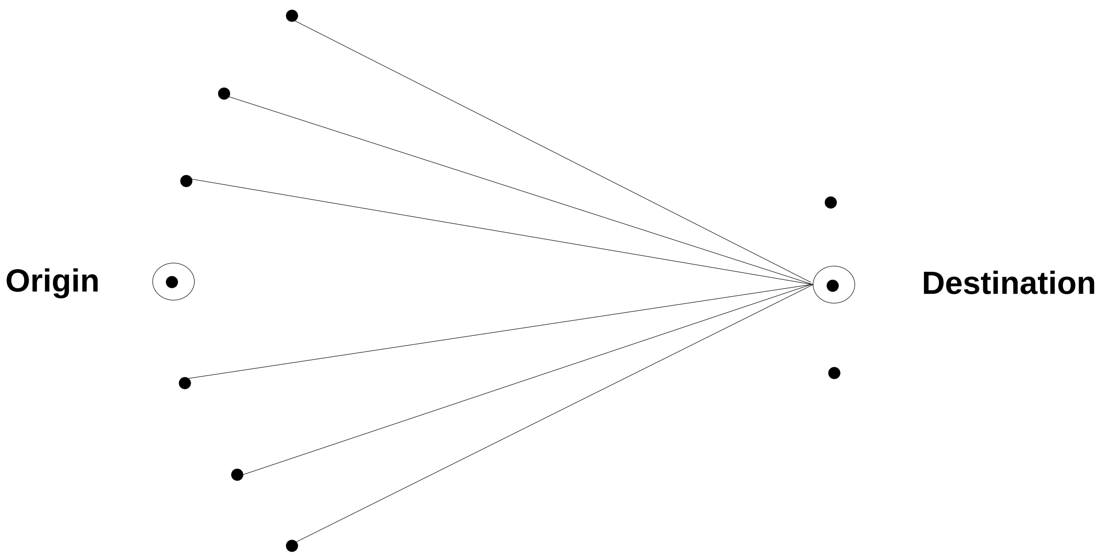
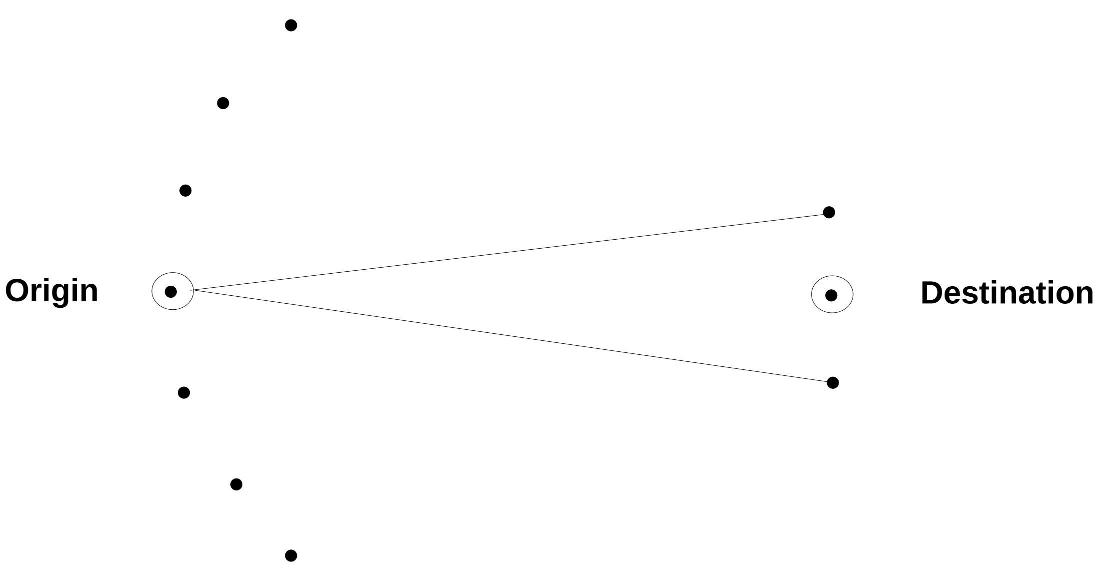


Figure 6 Origin-contiguity and destination-contiguity based rules



R_1 type



R_2 type

The second point of interest is that there is no need for similar numbers of points “around” origins and destinations. Indeed, Table 1 indicates clearly that, because of the spatial distribution of German cities – German cities are not uniformly and regularly distributed –, some regions have a higher density of cities, with the consequence that an origin-based criterion yields, under the distance rule, many more flows than a destination-based one, as illustrated in Figure 6.

Table 1. Impact Matrices, Germany 1985, 286 O-D pairs

R matrix with distance by car between 100 and 180 km	Number of lines				Characteristics of positive elements per line		
	Total	With positive elements					
		NONE	SOME				
			different	identical	MIN	MAX	MEAN
NORIC-O	286	69	189	28	1	27	6,05
NORIC-D	286	196	69	21	1	33	1,99
NORIC-OD	286	14	235	37	2	33	8,03

In fact, as can be seen in Table 1, the origin-based rule yields about twice as many contiguous neighbours than the destination-based rule per line: not only do more cities have neighbours but there are more neighbours.

4.2 Mode choice and the modal utility index

The model used to obtain the modal utility (4) is a mode share model: for each O-D pair t, the share of the mth mode is given by

$$sh(m)_t = \frac{e^{V_{m_t}}}{\sum_{p=1}^{M} e^{V_{p_t}}} \quad , \tag{20-A}$$

where the representative utility functions have the structure

$$V_{m_t} = \beta_{om} + \sum_k \beta_k X_{k_t}^{(\lambda_{x_k})} \tag{20-B}$$

and the X_k denote both network variables that vary across the modes and socioeconomic variables that do not, as well as trip purpose variables to account for the heterogeneity of the trips. We tried to maintain a specification as close as possible to that obtained with disaggregate 1979 Kontifern data (Mandel, Gaudry and Rothengatter, 1997) which had, for any individual i:

$$V_{m_i} = \beta_{o_m} + \sum_n \beta_n \, N_{n_i}^{(\lambda_{x_n})} + \sum_a \beta_a \, S_{a_i} + \sum_p \beta_p \, P_{p_i} \tag{21}$$

with	N =	cost, time, frequency
	S =	age class, sex, employment status, household size
	P =	business, private and vacation

and included only one λ_{x_n} in the optimal model. However, due to the nature of aggregate data, the following modifications to the specification (21) had to be made

N: Network variables normally appear in logit models (and also in many other classes of models) in a form that can be called an expenditure of time or money, namely:

$$\left[\begin{array}{lll} \text{Fare}_{OD_m} & : & \text{amount actually spent from the origin to the destination by mode } m; \\ \text{Travel time}_{OD_m} & : & \text{time spent travelling from the origin to the destination by mode } m. \end{array}\right] \quad (22)$$

It has long been found that, if the regression coefficients for these variables were not constrained to be equal across modes (as in (21) where the coefficients β_n do not vary with the mode m), i.e were specific to each mode and not generic, incorrect signs could be obtained. For this reason, explained in more details in Gaudry and Dagenais (1986), an alternate specification can be very useful. It consists of a rate form, namely:

$$\left[\begin{array}{llll} \text{Price}_{OD_m} & : & \text{Fare}_{OD_m}/\text{Distance}_{OD_m} & , \\ \text{Speed}_{OD_m} & : & \text{Distance}_{OD_m}/\text{Travel time}_{OD_m} & , \\ \text{Distance}_{OD_m} & & & , \end{array}\right] \quad (23)$$

where it is clear that the first two variables apply to units of distance, and can be thought of as money and time prices per unit, and the distance variable has a role similar to that of income in standard microeconomic demand system specifications that are of the form

$$q_m^d = (\text{price}_m, ..., \text{ income}) \quad (24)$$

where the quantity demanded of good $\underline{m}, q_m^d$, is explained by its price per unit (and also perhaps by prices of other goods) and income.

In this application to Germany with aggregate data for 1985, the very high collinearity between the Fare and Travel time variable for the car mode made it impossible to obtain expected signs for both of these variables at the same time except for some fragile cases. To obtain robust results, two possibilities remained. A first option would constrain the relative weights of Fare and Travel time, using the weights found with the 1979 disaggregate database, to define a “generalized cost” or impedance. This option, often used in practice to get around collinearity problems, has the difficulty that the weights are known to vary with the functional form of the variables – indeed our previous study has precisely shown that strong nonlinearities of the representative utility function yielded by far the most credible results. The other option would resort to the rate form (23), thus allowing for BCT on each variable. It was adopted and proved to be robust. The reader should note in interpreting the results that the distance term can be thought of as a basic or reference amount of time and money “income” that has to be spent going from O to D and that the unit prices affect (given the distance) the choice between the quantities of each mode that are purchased.

There are ways of exploring, with BCT, how one can nest (23) into (22): it is clear for instance that if the form of (23) is logarithmic, there should be a way of obtaining (22) by recombining terms and putting constraints on coefficients. However we did not have the resources to explore this further.

S_a: In an aggregate (market share) model, the format (2) is appropriate. The population variables are all defined as proportions of the total in the zone $(i \text{ or } j)$:

POP0014	:	Population aged from 0 to 14 / total population,
POP5064	:	Population aged from 50 to 64 / total population,
POPMALE	:	Number of men / total population,
TOTEMPL	:	Employees (total) / total population.

Although information was also available on the age group 15-49, the resulting variable, used in the auto and train functions, was quite collinear with the POPMALE and car price variables: this means that both 15-49 and 65 + age classes constitute the implicit reference category.

P_p: As the flows by trip purpose are known for all OD pairs, it is possible to construct variables describing the proportion of total trips made for each trip purpose on each O-D pair:

BUSINESS	:	Business trips by all modes / total trips,
PRIVATE	:	Private trips by all modes / total trips,

and we neglect to construct a Vacation variable: this trip purpose constitutes the implicit reference category. Reference category variables cannot be used simultaneously with the other category variables because they would sum up to 1 and be collinear with the regression constant.

Table 2 contains the results of the mode choice model, showing three different variants, one per column. The first part of the table contains own elasticities, as well as, for the underlying β regression coefficients (denoted as generic (GEN) – or common to the 3 modes – or specific (SPE) to each modal equation), the *t*-statistics computed conditionnally upon the values of the BCT indicated in the second part of the table. The third part of the table includes general statistics, notably the value of the log-likelihood of the observations. One may note

- functional form : there are large gains in log likelihood in moving away from the linear form: about 6 points in allowing one free BCT parameter and an additional 20 in allowing 3 more (one per network variable). However the *t*-statistics indicate that the price and speed BCT are mostly responsible for these large gains, not the distance and frequency BCT: in the case of distance, one cannot reject the logarithmic form $(\lambda = 0)$ and in the case of frequency one cannot reject the linear $(\lambda = 1)$ starting point.

  It is also noteworthy that allowing the BCT to adjust generally increases the statistical significance of the network variables, reduces that of the socioeconomic variables and has no overall impact on the trip purpose variables.

  The power of the BCT is also shown in the rail equation where one socioeconomic variable changes sign but is always significant!

- specific variables : Although we did not examine in detail and thoroughly compare the results obtained with specification (23) for different values of BCT in order to fine tune the estimates, the results appear to be generally reasonable, even for the very high rail price elasticity due to the negative BCT. Note that the elasticities are calculated as:

$$\eta(p_m, X_k) = \frac{\partial p_m}{\partial X_k} \frac{X_k}{p_m} = \beta_k(1 - p_m)\ X_k^{\lambda_{x_k}} \quad , \tag{25}$$

Table 2. Linear and Box-Cox Logit Share Models Germany 1985 (Symmetric Flows)

VARIANT		1	2	3
INDEPENDENT VARIABLE	BETA COEFFICIENT	OWN ELASTICITY (CONDITIONAL t)		
NETWORK		ALTERNATIVE: AIR		
PRICE-AIR	GEN	-1.93	-0.46	-3.88
		(-4.92)	(-2.47)	(-6.99)
SPEED-AIR	GEN	0.92	1.13	0.47
		(2.92)	(5.72)	(4.97)
DIST-AIR	GEN	-0.70	-0.05	-1.20
		(-2.64)	(-0.58)	(-4.34)
FREQ-AIR	GEN	0.27	0.06	0.06
		(4.50)	(6.67)	(6.99)
NETWORK		ALTERNATIVE: RAIL		
PRICE-AIR	GEN	-0.88	-0.03	-14.67
		(-4.92)	(-2.47)	(-6.99)
SPEED-AIR	GEN	0.59	0.30	0.05
		(2.92)	(5.72)	(4.97)
DIST-RAIL	GEN	-0.61	-0.04	-1.05
		(-2.64)	(-0.58)	(-4.34)
FREQ-RAIL	GEN	0.46	0.64	0.64
		(4.50)	(6.67)	(6.99)
SOCIOECONOMIC				
POP0014	SPE	-5.97	-5.96	-5.23
		(-4.39)	(-4.66)	(-3.83)
POP5064	SPE	0.23	-0.12	-0.22
		(5.90)	(5.55)	(4.07)
POPMALE	SPE	7.70	7.33	7.44
		(4.72)	(5.09)	(4.65)
TOTEMPL	SPE	0.93	0.81	0.84
		(2.70)	(3.29)	(2.87)
TRIP PURPOSE				
BUSINESS	SPE	-0.52	-0.51	-0.44
		(-9.71)	(-10.05)	(-10.98)
PRIVATE	SPE	-1.29	-1.19	-0.80
		(-0.59)	(1.36)	(0.16)

VARIANT		1	2	3
INDEPENDENT VARIABLE	BETA COEFFICIENT	OWN ELASTICITY (CONDITIONAL t)		
NETWORK		ALTERNATIVE: CAR		
PRICE-CAR	GEN	-0.28 (-4.92)	-0.01 (-2.47)	-3.56 (-6.99)
SPEED-CAR	GEN	0.20 (2.92)	0.12 (5.72)	0.03 (4.97)
DIST-CAR	GEN	-0.16 (-2.64)	-0.01 (-0.58)	-0.28 (-4.34)
SOCIOECONOMIC				
POP0014	SPE	1.46 (1.82)	1.32 (1.49)	1.18 (1.97)
POP5064	SPE	0.83 (7.70)	0.75 (7.78)	0.46 (5.86)
POPMALE	SPE	-0.92 (3.06)	-0.73 (3.53)	-1.07 (2.79)
TOTEMPL	SPE	-0.17 (0.73)	-0.11 (1.82)	-0.15 (1.00)
TRIP PURPOSE				
BUSINESS	SPE	-0.01 (-9.12)	-0.01 (-9.43)	-0.01 (-10.74)
PRIVATE	SPE	0.41 (5.47)	0.39 (7.90)	0.23 (3.68)
BOX-COX		PARAMETER VALUE (UNCONDITIONAL t with PAR=0) <UNCONDITIONAL t with PAR=1>		
LAMBDA	PRICE	1.00	3.88 (3.42) <2.54>	-2.20 (-4.10) <-5.96>
	SPEED	1.00	3.88 (3.42) <2.54>	6.39 (2.88) <2.43>
	DIST	1.00	3.88 (3.42) <2.54>	-0.15 (-0.25) <-1.89>
	FREQ	1.00	3.88 (3.42) <2.54>	3.94 (1.31) <0.98>
LOG-LIKELIHOOD		-709.96	-703.39	-683.90
R2 (overall)		0.58	0.58	0.61
NUMBER OF PAIRS		286	286	286

with all evaluations made at the sample means. One may understand that, if $\lambda_{x_k} < 0$, and quite large, as is the case for the price variable $\left(\hat{\lambda}_{x_k} = -2.20\right)$, a somewhat unusual value could occur that might require some probing.

Because of the large gains obtained with the four BCT in Variant 3, it was decided to use that variant to build the modal utility index for the generation-distribution model. The results for that model will therefore be conditional on the previous and separate estimates shown in Table 2, Col. 3. It would not at this point be reasonable to estimate the mode choice and distribution models simultaneously because of the computer time that would be required.

4.3 Generation-Distribution model

In addition to the U variable, constructed according to (4), the specification of the Generation-Distribution model shown in Table 3 or 4 required three more variables, all of which were constructed according to format (2): Population, Income (built using "Gross Value Product") and Size of zone (constructed to test for heteroskedasticity). The resulting specification is relatively simple, but adequate to demonstrate the usefulness of our chosen approach. More refined specifications that would incorporate, for instance, descriptors of the job mix (manufacturing, services, etc.) would be of certain interest but unlikely to modify deeply our results. The same may or may not be true of dummy variables to account for border effects: duly taking spatial correlation into account is probably insufficient to control for the fact that some cities are next to a border (the sea, the DDR, France, etc.) which conditions trip making either because it is not included in the model (trips abroad are excluded) or because reduced travel opportunities may increase flows to available opportunities: the number of contiguous neighbours affects the $\bar{R}$ matrix because, by the normalization procedure, lines where there are relatively many non-zero elements will give smaller weight to these observations than those containing fewer non-zero elements. However, this may not suffice to remove border effects. Only a more detailed study could, by adding border area dummy variables, answer the question.

The question of interest is clearly that of the impact of spatial correlation processes, which implicitly introduce spatial competition, on the elasticity of the utility of travel; moreover, we want to know whether this answer depends on the functional form of the model or on the way in which heteroskedasticity has been taken into account. To answer the question, we performed tests shown in Tables 3 and 4.

These tables, like Table 2, contain three sections: the first for elasticities of variables occurring in (7-A) or (7-B), the second for the BCT associated with such variables and for autocorrelation and proximity parameters of the (7-C) and (14) process; the third for general statistics. In both tables, the first six columns show the impact, starting with the usual multiplicative format (Col. 1), first of adding ρ under the assumption that proximity π is equal to one (Col. 2), then of allowing for distant neighbours (Col. 3), and further of estimating 1, 2 or 4 BCT (Columns 4, 5 and 6). The last 5 columns add heteroskedasticity to the cases of columns 2-6, and are labelled in the same sequence 7-11.

Table 3 results for the (NORIC-OD) autoregressive contiguous distributed process AR-C-D $(1, \infty, G(\pi_1))$ indicate the following

- functional form : the BCT is extremely powerful but, although the multiplicative case is easily rejected (as we note gains from Col. 3 to Col. 4 or 5 – or from Col. 8 to Col. 9 or 10 if heteroskedaskedasticity is considered –), there is no significant gain in going from 2 BCT to 4 BCT (Col. 5 vs 6; or Col. 10 vs 11);
- heteroskedasticity : although some heteroskedasticity seems present in the multiplicative form (Col. 2 vs 7), the use of the BCT progressively removes all trace (with SIZE of zone as explanatory variable) of heteroskedasticity;
- autocorrelation : the introduction of autocorrelation with the proximity parameter equal to 1 causes large gains (Col. 1 vs 2), and a value of 0,76 for ρ, which remains stable across variants;
- proximity : the introduction of the proximity parameter π (Col. 2 vs 3) does not cause very large gains in log likelihood but the estimated value (about 0.5 when BCT are present) is reasonable. In fact, as shown in Figure 7, the parameter π is quite different from 0 (in practice from 0.01, as $\tilde{R}$ in (14-D) vanishes for $\pi = 0$), which means that one can confidently reject the idea that distant neighbours matter much[8].

[8] For case 6 shown in Figure 7, values of the loglikelihood at 1 and 0,01 are -2996.83 and -3008.94; for our preferred case 5, they are, respectively -2999.14 and -3014.57. These exact values clearly give a better idea than the more approximate *t*-statistics shown in Tables 3 and 4.

Table 3. Generation-Distribution Models, First Order AR-C-D process Utility from Variant 3 (Table 2), Germany 1985 (Symmetric Flows)

VARIANT		1	2	3	4	5	6	7	8	9	10	11
CLASS		LOG	LOG+	LOG+	BC1+	BC2+	BC4+	LOG+	LOG+	BC1+	BC2+	BC4+
SUBCLASS			AU	AU+PR	AU+PR	AU+PR	AU+PR	AU+HG	AU+PR+HG	AU+PR+HG	AU+PR+HG	AU+PR+HG
INDEPENDENT VARIABLES							ELASTICITY (CONDITIONAL T)					
POPULATION	X1	1.37	1.51	1.49	1.33	1.41	1.36	1.51	1.49	1.33	1.41	1.36
		(14.35)	(17.52)	(16.52)	(12.28)	(12.89)	(13.32)	(17.51)	(16.47)	(12.36)	(12.92)	(13.27)
INCOME	X2	1.33	1.62	1.64	1.63	1.50	1.43	1.63	1.64	1.63	1.51	1.41
		(10.16)	(10.26)	(10.18)	(8.51)	(7.72)	(7.26)	(10.29)	(10.17)	(8.53)	(7.78)	(7.15)
UTILITY	X3	0.40	0.32	0.30	0.24	0.27	0.26	0.32	0.30	0.24	0.27	0.26
		(9.68)	(7.85)	(7.22)	(5.55)	(6.42)	(6.28)	(7.96)	(7.39)	(5.70)	(6.44)	(6.32)
HETEROSKEDASTICITY												
SIZE	Z1							0.00	0.00	0.03	0.00	-0.00
								(0.41)	(0.75)	(0.59)	(0.33)	(-0.42)
BOX-COX TRANSFORMATIONS							PARAMETER VALUE (UNCONDITIONAL T with PAR=0) <UNCONDITIONAL T with PAR=1>					
HETEROSKEDASTICITY												
LAMBDA	Z1							1.04	0.52	0.20	0.29	4.96
								(0.13)	(0.12)	(0.03)	(0.03)	(0.37)
								<0.00>	<-0.11>	<-0.14>	<-0.07>	<0.30>
DEPENDENT VARIABLE												
LAMBDA	Y	0.00	0.00	0.00	-0.37	-0.51	-0.45	0.00	0.00	-0.36	-0.51	-0.45
					(-3.42)	(-5.52)	(-4.88)			(-3.37)	(-5.49)	(-4.82)
					<-12.73>	<-16.30>	<-15.80>			<-12.63>	<-16.21>	<-15.64>
INDEPENDENT VARIABLES												
LAMBDA	X1	0.00	0.00	0.00	-0.37	0.45	0.66	0.00	0.00	-0.36	0.44	0.68
					(-3.42)	(2.86)	(2.29)			(-3.37)	(2.84)	(2.33)
					<-12.73>	<-3.51>	<-1.16>			<-12.63>	<-3.57>	<-1.08>
	X2	0.00	0.00	0.00	-0.37	0.45	1.57	0.00	0.00	-0.36	0.44	1.64
					(-3.42)	(2.86)	(1.90)			(-3.37)	(2.84)	(1.95)
					<-12.73>	<-3.51>	<0.69>			<-12.63>	<-3.57>	<0.76>
	X3	0.00	0.00	0.00	-0.37	0.45	0.33	0.00	0.00	-0.36	0.44	0.33
					(-3.42)	(2.86)	(1.46)			(-3.37)	(2.84)	(1.43)
					<-12.73>	<-3.51>	<-2.94>			<-12.63>	<-3.57>	<-2.96>
SPATIAL CORRELATION							PARAMETER VALUE (CONDITIONAL T with PAR=0) <CONDITIONAL T with PAR=1>					
O and D: DIST. (100-180 km)												
RHO (DIST_OD)			0.52	0.73	0.74	0.69	0.66	0.52	0.73	0.74	0.69	0.65
			(6.31)	(6.44)	(6.60)	(4.84)	(4.23)	(6.30)	(6.41)	(6.61)	(4.83)	(4.12)
PI (DIST_OD)			1.00	0.40	0.33	0.37	0.37	1.00	0.39	0.33	0.37	0.37
				(1.92)	(1.79)	(1.43)	(1.30)		(1.89)	(1.79)	(1.44)	(1.27)
				<-2.90>	<-3.60>	<-2.42>	<-2.19>		<-2.93>	<-3.67>	<-2.44>	<-2.19>
LOG-LIKELIHOOD		-3057.31	-3040.22	-3032.41	-3020.65	-2998.50	-2996.36	-3040.13	-3032.14	-3020.47	-2998.45	-2996.10
PSEUDO-(L)-R2 (adjusted for D.F.)		0.91	0.92	0.93	0.93	0.94	0.94	0.92	0.93	0.93	0.94	0.94
NUMBER OF PAIRS		286	286	286	286	286	286	286	286	286	286	286

Table 4. Generation-Distribution Models, Second Order AR-C-D process
Utility from Variant 3 (Table 2), Germany 1985 (Symmetric Flows)

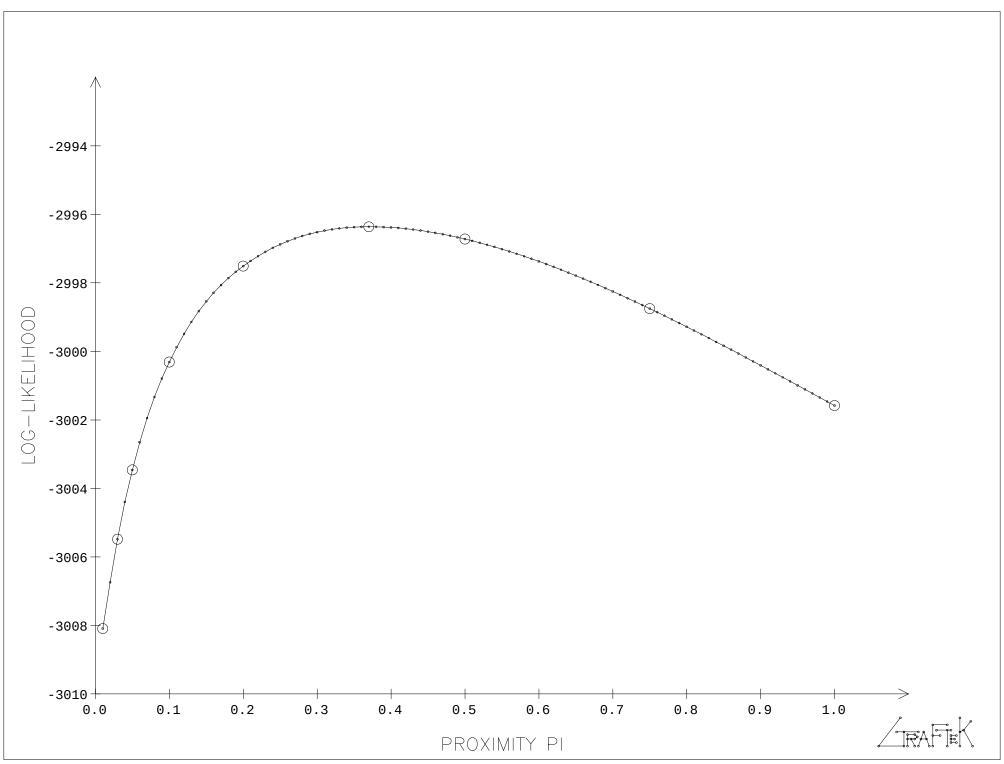
VARIANT		1	2	3	4	5	6	7	8	9	10	11
CLASS		LOG	LOG+	LOG+	BC1+	BC2+	BC4+	LOG+	LOG+	BC1+	BC2+	BC4+
SUBCLASS			AU	AU+PR	AU+PR	AU+PR	AU+PR	AU+HG	AU+PR+HG	AU+PR+HG	AU+PR+HG	AU+PR+HG
INDEPENDENT VARIABLES						ELASTICITY (CONDITIONAL T)						
POPULATION	X1	1.37	1.49	1.50	1.36	1.44	1.38	1.49	1.50	1.36	1.44	1.38
		(14.35)	(17.51)	(17.23)	(12.98)	(12.57)	(13.02)	(17.37)	(16.89)	(12.92)	(12.55)	(12.84)
INCOME	X2	1.33	1.57	1.61	1.58	1.50	1.37	1.58	1.62	1.59	1.48	1.34
		(10.16)	(10.13)	(9.81)	(8.20)	(7.51)	(7.07)	(10.04)	(9.64)	(8.22)	(7.45)	(6.87)
UTILITY	X3	0.40	0.34	0.34	0.28	0.31	0.30	0.34	0.34	0.29	0.32	0.30
		(9.68)	(8.17)	(8.05)	(6.45)	(7.18)	(7.07)	(8.18)	(8.05)	(6.47)	(7.24)	(7.11)
HETEROSKEDASTICITY												
SIZE	Z1							0.01	0.05	0.13	-0.00	-0.00
								(0.38)	(0.55)	(0.35)	(-0.45)	(-0.76)
BOX-COX TRANSFORMATIONS						PARAMETER VALUE (UNCONDITIONAL T with PAR=0) <UNCONDITIONAL T with PAR=1>						
HETEROSKEDASTICITY												
LAMBDA	Z1							0.26	0.08	-0.15	5.66	5.15
								(0.03)	(0.01)	(-0.02)	(0.39)	(0.62)
								<-0.09>	<-0.16>	<-0.12>	<0.32>	<0.50>
DEPENDENT VARIABLE												
LAMBDA	Y	0.00	0.00	0.00	-0.33	-0.53	-0.45	0.00	0.00	-0.33	-0.53	-0.44
					(-3.20)	(-5.28)	(-4.65)			(-3.16)	(-5.27)	(-4.57)
					<-12.81>	<-15.33>	<-15.04>			<-12.72>	<-15.25>	<-14.88>
INDEPENDENT VARIABLES												
LAMBDA	X1	0.00	0.00	0.00	-0.33	0.46	0.70	0.00	0.00	-0.33	0.46	0.74
					(-3.20)	(3.22)	(2.32)			(-3.16)	(3.26)	(2.41)
					<-12.81>	<-3.79>	<-0.98>			<-12.72>	<-3.81>	<-0.85>
	X2	0.00	0.00	0.00	-0.33	0.46	1.97	0.00	0.00	-0.33	0.46	2.05
					(-3.20)	(3.22)	(2.28)			(-3.16)	(3.26)	(2.36)
					<-12.81>	<-3.79>	<1.12>			<-12.72>	<-3.81>	<1.21>
	X3	0.00	0.00	0.00	-0.33	0.46	0.34	0.00	0.00	-0.33	0.46	0.33
					(-3.20)	(3.22)	(1.74)			(-3.16)	(3.26)	(1.71)
					<-12.81>	<-3.79>	<-3.40>			<-12.72>	<-3.81>	<-3.45>
SPATIAL CORRELATION						PARAMETER VALUE (CONDITIONAL T with PAR=0) <CONDITIONAL T with PAR=1>						
O: DIST. (100-180 km)												
RHO (DIST_O)			0.39	0.46	0.42	0.40	0.34	0.39	0.46	0.42	0.41	0.35
			(3.83)	(3.31)	(2.75)	(2.49)	(1.97)	(3.79)	(3.20)	(2.68)	(2.52)	(2.06)
PI (DIST_O)			1.00	0.67	0.68	0.75	0.82	1.00	0.65	0.66	0.74	0.79
				(1.52)	(1.38)	(1.53)	(1.47)		(1.49)	(1.36)	(1.52)	(1.45)
				<-0.75>	<-0.66>	<-0.50>	<-0.33>		<-0.79>	<-0.69>	<-0.53>	<-0.38>
D: DIST. (100-180 km)												
RHO (DIST_D)			0.36	0.40	0.43	0.27	0.27	0.36	0.39	0.42	0.28	0.26
			(2.82)	(2.18)	(2.35)	(1.35)	(1.29)	(2.74)	(2.10)	(2.31)	(1.21)	(1.11)
PI (DIST_D)			1.00	0.68	0.53	0.97	1.00	1.00	0.71	0.54	0.71	0.79
				(0.78)	(0.64)	(0.78)	(0.77)		(0.80)	(0.65)	(0.50)	(0.52)
				<-0.37>	<-0.57>	<-0.02>	<0.00>		<-0.33>	<-0.55>	<-0.20>	<-0.14>
LOG-LIKELIHOOD		-3057.31	-3043.34	-3042.11	-3032.23	-3007.04	-3003.44	-3043.27	-3041.98	-3032.17	-3006.62	-3002.63
PSEUDO-(L)-R2 (adjusted for D.F.)		0.91	0.92	0.92	0.93	0.94	0.94	0.92	0.92	0.93	0.94	0.94
NUMBER OF PAIRS		286	286	286	286	286	286	286	286	286	286	286

• specific results : the most important impact of the introduction of autocorrelation is the large reduction of the elasticity of the utility index (from 0.40 to 0.24), as one would expect if an IIA consistent model understated the impact of substitute destinations. A second result is an increase in the elasticity of the Population and Income terms (Col. 1 vs 2). In this case, the elasticities of the expected value of T_{ij} are defined as

$$\eta(T, X_k) = \frac{1}{N} \sum_{t=1}^{N} \left(\frac{\partial\, E(T_t)}{\partial\, X_{k_t}} \cdot \frac{X_{k_t}}{E(T_t)} \right) \ , \tag{26}$$

where the formulas for the derivatives are supplied in detail in Liem and Gaudry (1994a) for all values of λ_y, λ_{x_k} and for the specifications where Z_m is used in (7-B) – in which case we want the elasticity of T to Z_m, as found in the heteroskedasticity subsection in the first part of both Table 3 and 4. A third result is that the logarithmic form of the utility term U should probably be rejected, as one compares Column 5 to column 6.

Figure 7. Behaviour of log-likelihood of variant 6 (Table 3) over complete range of proximity parameter π



In Table 4, two significant differences should be noted. The first is that one would be inclined to retain the model with 4 BCT instead of the model with 2 BCT – Column 6 instead of Column 5. The second is that, although the joint use of ρ_1 and ρ_2 is clearly worthwhile, (Col. 1 vs 2), the introduction of π_1 and π_2 is not; moreover, π_2 converges to 1, implying that, under (NORIC-D), only contiguous neighbours matter, a result that is not surprising in view of the sparseness of the residue impact matrix R_2 symbolized in Figure 6 and described in Table 1: sparseness kills any effect of distant neighbours.

However, using two proximity structures simultaneously may be more fragile than using only one, especially if the values of ρ_1 and ρ_2 are quite close to each other – if they were equal, one would implicitly be adding matrices $\tilde{R}_1$ and $\tilde{R}_2$ and implicitly violating the normalization rule for the resulting matrix. To explore this point further, we made tests shown in Table A.1 of the Appendix, using only multiplicative models. As the value of π_2 always converged to 1, irrespective of whether ρ_1 and π_1 were simultaneously estimated, we are confident that the absence of influence of distant neighbours defined by (NORIC-D) reflects the use of that contiguity rule, rather than an underlying difficulty of simultaneously estimating two distributed lag structures.

The conclusion to be drawn from these two series of tests is that the introduction of spatial competition through an autoregressive contiguous distributed process works as expected in the sense that the elasticity of the modal utility index (or inclusive price) falls as expected where autocorrelation is positive and accounts for the influence of substitutes, but that the additional role of the proximity parameter in determining the relative influence of close and distant neighbours very much depends on the specific spatial features captured by the residue impact matrix construction rules.

5. Implications of the approach: the Quasi-Direct Format (QDF)

In effect, one way to view the application is to combine the generation-distribution equation (1) and mode choice equation (24) as a product that explains trip demand by mode T_m namely, neglecting the subscripts for origin-destination pairs:

$$T_m = T \cdot p_m \quad , \tag{27}$$

which is the quasi-direct format proper (QDF) in which the dot separates total demand and market share parts

$$T = g^d(\{A_s\}, U) \tag{28}$$

and

$$p_m = U_m/U \quad , \qquad m = 1, ..., M \tag{29}$$

where

$$U_m = u_m(\{N_n\}, \{A_s\}) \quad , \tag{30}$$

$$U = (U_1 + ... + U_M) \quad , \tag{31}$$

and the sets $\{A_s\}$ and $\{N_n\}$ respectively denote socioeconomic $(s = 1, ..., S)$ and network variables $(n = 1, ..., N)$, and $g^d(\cdot)$ and $u_m(\cdot)$ are functions like (7-A to 7-C) and (3) + (20-B).

Using QDF means that the effect of any variable X_k on T_m, trip demand by mode, can be decomposed between its impact on p_m, mode share, and its impact on T, total demand irrespective of mode. If this effect is expressed in terms of elasticities η, it is easy to show that, because QDF is a product, we have

$$[\eta \text{ of Mode}] \equiv [\eta \text{ of Total}] + [\eta \text{ of Share}]$$

$$\text{or} \tag{32}$$

$$\eta(T_m, X_k) \;=\; \eta(T, X_k) \;+\; \eta(p_m, X_k) \;,$$

where three interesting cases arise according to whether X_k

Case L) is in the Total model, but not in the share model – X_k is in effect an A_a variable in (5);

Case S) is in the Share model (and consequently appears also in the modal utility term U of (5));

Case LS) is in both the Total model as an A_a variable and in the Share model (and consequently also appears in the modal utility term U of (5));

namely, explicating (32):

$$\eta(T_m, X_k)_L = \eta\big(T, X_{k,A}\big) \tag{33-L}$$

$$\eta(T_m, X_k)_S = \eta(T, U) \cdot \eta\big(U, X_{k,U}\big) + \eta(p_m, X_k) \tag{33-S}$$

$$\eta(T_m, X_k)_{LS} = \underbrace{\eta\big(T, X_{k,A}\big)}_{\text{(F)}} + \underbrace{\overset{\text{(A)}}{\eta(T, U)} \cdot \overset{\text{(B)}}{\eta\big(U, X_{k,U}\big)}}_{\text{(E)}} + \underbrace{\eta(p_m, X_k)}_{\text{(D)}} \tag{33-LS}$$

(33-QDF)

Naturally, we are especially interested in the modal utility, or inclusive value, term U for which (32) is written more explicitly as (33-S), a decomposition that makes explicit the two elasticities already shown in tables above, namely (D) in the mode choice model and (A) or (B) in the generation-distribution model. In addition, the decomposition presents the opportunity to compute (E) and (F), as well as what we shall call the DIVERSION RATE, an interesting statistic obtained from the explication of adjustments in T_m to distinguish how much comes from a variation of total demand (the INDUCTION RATE) and from a diversion to or from other modes (the DIVERSION RATE). We examine these in turn.

5.1 Explaining trip demand by mode

A question that immediately arises when we look at (33) is whether there can be a problem of consistency among the various parts of the formula that explains trip demand by mode. Is it possible for instance that the mode share variation subsequent to a change in X_k be offset by the generation-distribution part in such a way as to imply a smaller impact on trips by mode (F) than the mode shift (D) requires?

We have already pointed out that, because estimates of (B) were obtained conditionally upon estimates of (D) – an hence of (C) –, the sequential estimation procedure is inefficient (but computationally tractable) and may also be inconsistent: Laferrière (1988) has shown, in a single-mode model of air travel demand where itinerary choices were explained, that joint estimation of (A)-(B)-[(C)-(D)] was efficient in the sense of yielding a better fit, but has not analyzed statistical consistency issues.

The intuitive meaning of the consistency question is mathematical. To answer it, note that (B) is assumed to be positive and that (C) and (D) are assumed to be always of the same sign. It follows that (F) is at least as large as (D). Consider the following cases

i) if total demand is unresponsive to modal utility, then (B)=0 and the impact on trips by mode equals the impact on the modal share: (F)=(D);

ii) if modes are perfect complements (not a very realistic case), then (D)=0 and, assuming one can make sense of the case, |(F)| is at least as large as |(D)|;

iii) only if (C) and (D) were opposite signs could inconsistency arise in practice. This would require the mode choice model to allow for very strong cross-effects. This would occur if as e^{V_1} say decreased, $e^{V_2} + ... + e^{V_n}$ simultaneously increased more, so that the sum $U = \sum_m e^{V_m}$ increased as e^{V_1} decreased. This is conceptually possible in a model, such as the generalized Box-Cox Logit, where all network characteristics can be included in all V_i utility functions; however, as in systems of demand equations cross (off-diagonal) effects are normally weaker than own (diagonal) effects – otherwise one would ask questions about the definition of the goods considered –, this is unlikely to occur in general and impossible in a standard Box-Cox logit model. It could arise in a generalized Box-Cox logit model or in other specifications that allow all network characteristics of all modes to appear in each utility functions of each mode.

5.2 Allocation of change in trip demand by mode: diversion and induction

Consider the "stylized hypothesis" concerning the TGV line from Paris to Lyon[9] shown in Table 5. The QDF format allocates the increased traffic on the link between generation-distribution and mode choice. Granted that (F) is at least as large as (D), how can the increased flows shown in Table 5 be understood?

Table 5. Paris-Lyon Stylized Hypothesis*

Mode	Air	Train	Car	Total
Before TGV opening	30	30	40	100
After TGV opining	10	75	35	120

* The numbers are not real and were presented for the sake of argument by Olivier Morellet of INRETS.

In effect, the Quasi-Direct Format (QDF) distinguishes between the impact of a variable on mode choice and its impact on the total demand, according to the 3 cases of (33). The benefit of this procedure is to concentrate the variables that are most relevant for either problem in the appropriate part of the formula. It can be conjectured that the recent tendency to increase the number of socioeconomic variables in mode choice models is largely due to a disproportionate effort to study the mode choice problem at the expense of the generation-distribution model, and that a more balanced approach is necessary. Eventual joint estimation of both models should induce a migration of some socioeconomic variables from the mode choice to the generation-distribution piece.

A useful way of understanding the changes in rail demand represented in Table 5, and for that matter the change in the demand for any mode, T_m, resulting from changes in the k^{th} characteristic of the m^{th} mode, X_k^m, is to view it, following Liem and Gaudry (1994b), as a component of

$$\frac{\partial T}{\partial X_k^m} = \frac{\partial T_m}{\partial X_k^m} + \sum_{j \neq m} \frac{\partial T_j}{\partial X_k^m} \quad , \tag{34-A}$$

9 Actual before and after (1993) market shares for the AVE Madrid-Seville 470km link were: Iberia Airline 18 %, 7 %; Train: 20 %, 44 % and car 51 %, 39 % (The Financial Times, 15/3/94).

namely as the result of the decomposition of a change in total demand T into an effect on mode m, $T_{m,}$ and a remaining effect on the other modes T_j, (with $j \neq m$). This tautology can, along the lines suggested by Laferrière (1992) to obtain in the usual way the elasticities, be transformed to yield, after multiplication of all terms by $\left(X_k^m/T\right)$ and multiplication of the first term on the right hand side by (T_m/T_m) and of the second term by (T_j/T_j):

$$\eta_{T,X_k^m} = p_m \eta_{T_m,X_k^m} + \sum_{j \neq m} p_j \eta_{T_j,X_k^m} \quad , \tag{34-B}$$

$$\frac{\eta_{T,X_k^m}}{p_m \eta_{T_m,X_m^k}} = 1 + \sum_{j \neq m} \frac{p_j \eta_{T_j,X_k^m}}{p_m \eta_{T_m,X_k^m}} \quad , \tag{34-C}$$

$$(IR) = 1 + (DR) \tag{34-D}$$

which defines a diversion or substitution rate DR, and its complement the induction rate IR. The former is

$$DR(T_m, X_k^m) = \frac{\eta_{T,X_k^m}}{\eta_{T_m,X_k^m} p_m} - 1 \quad . \tag{35}$$

It captures the diversion or rate at which modified demand for mode m results from diversion to/from other modes, as opposed to changes in total demand. However, as the signs of the elasticities in (35) clearly matter, the diversion rate does not neatly fluctuate between -1 and 0. Let us discuss its behaviour, starting with the simplest case.

If total demand is insensitive to X_k^m, the diversion rate is -1: the change in modal demand arises completely from a substitution among the modes as one more trip by mode m implies one trip fewer by the other modes. Similarly, if the share elasticity $\eta\left(p_m, X_k^m\right)$ is 0, we must have by (32-33) that the modal and total elasticities are equal; if it is also the case that the share of mode m is equal to 1, then the diversion rate equals 0.

In the "usual case" of a network variable belonging only to the representative utility function of its own mode (X_k^m is only in V_m), a diversion rate of -0,80 simply means that 80 % of the effect comes from a diversion from other modes and 20 % from a modification of total demand, because the induction rate $IR = 1 + DR$. In the more complex case of a network variable belonging to its own and other representative utility functions – this may occur in a generalized Box-Cox logit model or in other models of a more complicated fabric than the standard Box-Cox logit model – the diversion rate could be positive. The difference between these cases is that, if the modes behave as substitutes, $DR < 0$; if they behave as complements, and both diversion and induction rates reinforce each other, then we may have $DR > 0$. In the more usual case of a socioeconomic variable appearing in more than one utility function, the diversion rate can clearly have any value as, for instance, the modification in the demand for mode m may well imply much more than a 1 to 1 diversion: a rate of -3 would mean that modifications of X_k^m result in relatively large impacts on modal demand T_m, perhaps because the total elasticity $\eta_{_T}$ in (35) is relatively high, or the market share of the mode, p_m, is relatively small. Indeed the formula makes it clear that smaller modes will naturally have higher diversion rates to/from other modes, and that models with lower $\eta_{_T}$ will have diversion rates closer to -1. The diversion rate takes into account the sign of the various effects.

By contrast, Laferrière's "diversion index" is obtained by considering the absolute value of $\partial T_m / \partial X_k^m$ in (34-A). This yields

$$DI(T_m, X_k^m) = 1 - \frac{\eta_{_{T,X_k^m}}}{\eta_{_{T_m,X_k^m}} p_m} \tag{36}$$

which is restricted between 0 and 1 for the "usual case" mentioned above but is not in the more general case of a network or socioeconomic variable belonging to many representative utility functions. Laferrière also points out that, if the total demand model is multiplicative, diversion can be expressed more simply in terms of $\hat{\beta}_U$, the elasticity of the modal utility term, which would imply for our diversion rate

$$DR(T_m, X_k^m) = \frac{\left(\hat{\beta}_U - 1\right)(1 - \hat{p}_m)}{1 + \left(\hat{\beta}_U - 1\right)\hat{p}_m} \quad . \tag{37}$$

We also note that, if observed shares $\hat{p}_m$ are substituted for estimated p_m shares in (37), a "quick and dirty" measure of the diversion rate can be easily computed for this special case.

5.3 <u>Derived modal elasticities and diversion rates</u>

In Table 6, we have regrouped the principal results of interest within the quasi-direct format QDF defined by (33-QDF) and (35). In the first part of the table, one finds elasticities (A) and (B); in the second part, one finds (C)-(F), as well as the Diversion Rate. More formally

$$
\left.\begin{array}{llll}
\textbf{\textit{Level}} & \text{(A)} & : & \eta(T,X) \\
 & & & \text{(t-statistic of underlying coefficient)} \\
 & \text{(B)} & : & \eta(T,U) \\
 & & & \text{(t-statistic of underlying coefficient)} \\
\textbf{\textit{Share}} & \text{(C)} & : & \eta(U,X) \\
 & \text{(D)} & : & \eta(p_m,X) \\
 & & & \text{(t-statistic of underlying coefficient)} \\
 & \text{(E)} & : & \eta(T) = (A) + [(B)\cdot(C)] \\
 & \text{(F)} & : & (D) + (E) \\
 & \text{D.R.} & : & [(E)/(F)\cdot p_m] - 1
\end{array}\right] \quad (38)
$$

As it is important to keep the market shares in mind, the table also lists them for our sample.

The cases selected for Germany are chosen to give a perspective. Column 1 starts with the usual multiplicative model. The next three columns (Columns 2-4) successively consider the impact of adding autocorrelation, proximity and BCT under R_1 = (NORIC-OD) and the last three (Columns 5-7) under R_1 = (NORIC-O) and R_2 = (NORIC-D). The reader should also note that, as the mode choice model is the same for all 7 cases, the lines for (C) and (D) are naturally identical across columns; similarly, within a column, diversion rates for a given alternative are the same for all network variables[10].

The most interesting result in Table 6 is that, as expected, the introduction of spatial correlation not only reduces the elasticity of the utility term from 0,40 in the first column to a much lower value in other columns, but also makes the diversion ratios closer to -1 for network variables. This simply means that the effects on modal demand T_m come relatively more from diversion than from induction, a perfectly reasonable result.

[10] This is due to the fact that the variable appears only in one alternative and would also hold if the regression coefficients in the mode choice model had been specific instead of generic or equal across alternatives.

Table 6. Share (Sm), Total (T) and Modal (Tm) Elasticities; Diversion Rates (D.R.)
Models from Table 3 and Table 4. Germany 1985 (Symmetric Flows)

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC4+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC4+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
LEVEL								
POPULATION	EL(T,X)	1.373	1.509	1.487	1.363	1.494	1.504	1.381
	(t)	(14.25)	(17.52)	(16.52)	(13.32)	(17.51)	(17.23)	(13.02)
INCOME	EL(T,X)	1.328	1.621	1.635	1.426	1.570	1.606	1.365
	(t)	(10.16)	(10.26)	(10.18)	(7.26)	(10.13)	(9.81)	(7.07)
UTILITY	EL(T,U)	0.400	0.321	0.297	0.259	0.340	0.337	0.299
	(t)	(9.68)	(7.85)	(7.22)	(6.28)	(8.17)	(8.05)	(7.07)
SHARE								
ALTERNATIVE (MEAN = 0.111): AIR								
PRICE-AIR	EL(U)	-0.203	-0.203	-0.203	-0.203	-0.203	-0.203	-0.203
	EL(Sm)	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875	-3.875
	(t)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)
	EL(T)	-0.081	-0.065	-0.060	-0.053	-0.069	-0.068	-0.061
	EL(Tm)	-3.957	-3.941	-3.936	-3.928	-3.944	-3.944	-3.936
	D.R.	-0.815	-0.851	-0.862	-0.879	-0.842	-0.843	-0.861
SPEED-AIR	EL(U)	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	EL(Sm)	0.470	0.470	0.470	0.470	0.470	0.470	0.470
	(t)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)
	EL(T)	0.010	0.008	0.007	0.006	0.008	0.008	0.007
	EL(Tm)	0.480	0.478	0.477	0.476	0.478	0.478	0.477
	D.R.	-0.815	-0.851	-0.862	-0.879	-0.842	-0.843	-0.861
DIST-AIR	EL(U)	-0.063	-0.063	-0.063	-0.063	-0.063	-0.063	-0.063
	EL(Sm)	-1.200	-1.200	-1.200	-1.200	-1.200	-1.200	-1.200
	(t)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)
	EL(T)	-0.025	-0.020	-0.019	-0.016	-0.021	-0.021	-0.019
	EL(Tm)	-1.225	-1.220	-1.219	-1.217	-1.222	-1.221	-1.219
	D.R.	-0.815	-0.851	-0.862	-0.879	-0.842	-0.843	-0.861
FREQ-AIR	EL(U)	0.003	0.003	0.003	0.003	0.003	0.003	0.003
	EL(Sm)	0.059	0.059	0.059	0.059	0.059	0.059	0.059
	(t)	(6.99)	(6.99)	(6.99)	(6.99)	(6.99)	(6.99)	(6.99)
	EL(T)	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	EL(Tm)	0.061	0.060	0.060	0.060	0.061	0.060	0.060
	D.R.	-0.815	-0.851	-0.862	-0.879	-0.842	-0.843	-0.861
ALTERNATIVE (MEAN = 0.223): RAIL								
PRICE-RAIL	EL(U)	-3.000	-3.000	-3.000	-3.000	-3.000	-3.000	-3.000
	EL(Sm)	-14.675	-14.675	-14.675	-14.675	-14.675	-14.675	-14.675
	(t)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)
	EL(T)	-1.200	-0.963	-0.891	-0.777	-1.020	-1.011	-0.897
	EL(Tm)	-15.874	-15.637	-15.565	-15.451	-15.694	-15.685	-15.571
	D.R.	-0.661	-0.724	-0.743	-0.774	-0.708	-0.711	-0.741

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC4+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC4+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
SPEED-RAIL	EL(U)	0.011	0.011	0.011	0.011	0.011	0.011	0.011
	EL(Sm)	0.054	0.054	0.054	0.054	0.054	0.054	0.054
	(t)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)
	EL(T)	0.004	0.004	0.003	0.003	0.004	0.004	0.003
	EL(Tm)	0.059	0.058	0.058	0.057	0.058	0.058	0.058
	D.R.	-0.661	-0.724	-0.743	-0.774	-0.708	-0.711	-0.741
DIST-RAIL	EL(U)	-0.215	-0.215	-0.215	-0.215	-0.215	-0.215	-0.215
	EL(Sm)	-1.053	-1.053	-1.053	-1.053	-1.053	-1.053	-1.053
	(t)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)
	EL(T)	-0.086	-0.069	-0.064	-0.056	-0.073	-0.073	-0.064
	EL(Tm)	-1.139	-1.122	-1.117	-1.109	-1.126	-1.125	-1.117
	D.R.	-0.661	-0.724	-0.743	-0.774	-0.708	-0.711	-0.741
FREQ-RAIL	EL(U)	0.132	0.132	0.132	0.132	0.132	0.132	0.132
	EL(Sm)	0.644	0.644	0.644	0.644	0.644	0.644	0.644
	(t)	(6.99)	(6.99)	(6.99)	(6.99)	(6.99)	(6.99)	(6.99)
	EL(T)	0.053	0.042	0.039	0.034	0.045	0.044	0.039
	EL(Tm)	0.697	0.687	0.684	0.679	0.689	0.689	0.684
	D.R.	-0.661	-0.724	-0.743	-0.774	-0.708	-0.711	-0.741
POP0014	EL(U)	0.688	0.688	0.688	0.688	0.688	0.688	0.688
	EL(Sm)	-5.226	-5.226	-5.226	-5.226	-5.226	-5.226	-5.226
	(t)	(-3.83)	(-3.83)	(-3.83)	(-3.83)	(-3.83)	(-3.83)	(-3.83)
	EL(T)	0.275	0.221	0.204	0.178	0.234	0.232	0.206
	EL(Tm)	-4.951	-5.005	-5.021	-5.048	-4.992	-4.994	-5.020
	D.R.	-1.250	-1.198	-1.183	-1.159	-1.210	-1.208	-1.184
POP5064	EL(U)	6.518	6.518	6.518	6.518	6.518	6.518	6.518
	EL(Sm)	-0.221	-0.221	-0.221	-0.221	-0.221	-0.221	-0.221
	(t)	(4.07)	(4.07)	(4.07)	(4.07)	(4.07)	(4.07)	(4.07)
	EL(T)	2.607	2.092	1.936	1.688	2.216	2.197	1.949
	EL(Tm)	2.386	1.871	1.715	1.467	1.995	1.975	1.728
	D.R.	3.905	4.019	4.068	4.165	3.986	3.991	4.063
POPMALE	EL(U)	8.527	8.527	8.527	8.527	8.527	8.527	8.527
	EL(Sm)	7.443	7.443	7.443	7.443	7.443	7.443	7.443
	(t)	(4.65)	(4.65)	(4.65)	(4.65)	(4.65)	(4.65)	(4.65)
	EL(T)	3.411	2.737	2.533	2.208	2.899	2.874	2.550
	EL(Tm)	10.853	10.180	9.975	9.651	10.342	10.316	9.992
	D.R.	0.411	0.207	0.140	0.027	0.258	0.250	0.145
TOTEMPL	EL(U)	0.518	0.518	0.518	0.518	0.518	0.518	0.518
	EL(Sm)	0.845	0.845	0.845	0.845	0.845	0.845	0.845
	(t)	(2.87)	(2.87)	(2.87)	(2.87)	(2.87)	(2.87)	(2.87)
	EL(T)	0.207	0.166	0.154	0.134	0.176	0.174	0.155
	EL(Tm)	1.052	1.011	0.998	0.979	1.021	1.019	0.999
	D.R.	-0.116	-0.262	-0.309	-0.385	-0.226	-0.231	-0.305
BUSINESS	EL(U)	-1.660	-1.660	-1.660	-1.660	-1.660	-1.660	-1.660
	EL(Sm)	-0.441	-0.441	-0.441	-0.441	-0.441	-0.441	-0.441
	(t)	(-10.98)	(-10.98)	(-10.98)	(-10.98)	(-10.98)	(-10.98)	(-10.98)
	EL(T)	-0.664	-0.533	-0.493	-0.430	-0.564	-0.559	-0.496
	EL(Tm)	-1.105	-0.974	-0.934	-0.871	-1.005	-1.000	-0.937
	D.R.	1.698	1.456	1.370	1.216	1.520	1.510	1.377

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC4+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC4+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
PRIVATE	EL(U)	0.846	0.846	0.846	0.846	0.846	0.846	0.846
	EL(Sm)	-0.797	-0.797	-0.797	-0.797	-0.797	-0.797	-0.797
	(t)	(0.16)	(0.16)	(0.16)	(0.16)	(0.16)	(0.16)	(0.16)
	EL(T)	0.338	0.271	0.251	0.219	0.287	0.285	0.253
	EL(Tm)	-0.458	-0.525	-0.545	-0.578	-0.509	-0.512	-0.544
	D.R.	-4.312	-3.320	-3.067	-2.702	-3.535	-3.500	-3.087

ALTERNATIVE (MEAN = 0.667): CAR

		1	2	3	4	5	6	7
PRICE-CAR	EL(U)	-12.648	-12.648	-12.648	-12.648	-12.648	-12.648	-12.648
	EL(Sm)	-3.556	-3.556	-3.556	-3.556	-3.556	-3.556	-3.556
	(t)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)	(-6.99)
	EL(T)	-5.059	-4.060	-3.756	-3.276	-4.300	-4.262	-3.782
	EL(Tm)	-8.615	-7.616	-7.312	-6.832	-7.856	-7.818	-7.338
	D.R.	-0.119	-0.200	-0.229	-0.281	-0.179	-0.182	-0.227
SPEED-CAR	EL(U)	0.094	0.094	0.094	0.094	0.094	0.094	0.094
	EL(Sm)	0.026	0.026	0.026	0.026	0.026	0.026	0.026
	(t)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)	(4.97)
	EL(T)	0.037	0.030	0.028	0.024	0.032	0.032	0.028
	EL(Tm)	0.064	0.056	0.054	0.051	0.058	0.058	0.054
	D.R.	-0.119	-0.200	-0.229	-0.281	-0.179	-0.182	-0.227
DIST-CAR	EL(U)	-1.008	-1.008	-1.008	-1.008	-1.008	-1.008	-1.008
	EL(Sm)	-0.283	-0.283	-0.283	-0.283	-0.283	-0.283	-0.283
	(t)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)	(-4.34)
	EL(T)	-0.403	-0.324	-0.299	-0.261	-0.343	-0.340	-0.301
	EL(Tm)	-0.687	-0.607	-0.583	-0.545	-0.626	-0.623	-0.585
	D.R.	-0.119	-0.200	-0.229	-0.281	-0.179	-0.182	-0.227
POP0014	EL(U)	0.688	0.688	0.688	0.688	0.688	0.688	0.688
	EL(Sm)	1.180	1.180	1.180	1.180	1.180	1.180	1.180
	(t)	(1.97)	(1.97)	(1.97)	(1.97)	(1.97)	(1.97)	(1.97)
	EL(T)	0.275	0.221	0.204	0.178	0.234	0.232	0.206
	EL(Tm)	1.455	1.401	1.385	1.358	1.414	1.412	1.386
	D.R.	-0.716	-0.763	-0.779	-0.803	-0.752	-0.754	-0.777
POP5064	EL(U)	6.518	6.518	6.518	6.518	6.518	6.518	6.518
	EL(Sm)	0.463	0.463	0.463	0.463	0.463	0.463	0.463
	(t)	(5.86)	(5.86)	(5.86)	(5.86)	(5.86)	(5.86)	(5.86)
	EL(T)	2.607	2.092	1.936	1.688	2.216	2.197	1.949
	EL(Tm)	3.071	2.556	2.399	2.152	2.680	2.660	2.412
	D.R.	0.274	0.228	0.210	0.177	0.241	0.239	0.212
POPMALE	EL(U)	8.527	8.527	8.527	8.527	8.527	8.527	8.527
	EL(Sm)	-1.075	-1.075	-1.075	-1.075	-1.075	-1.075	-1.075
	(t)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)	(2.79)
	EL(T)	3.411	2.737	2.533	2.208	2.899	2.874	2.550
	EL(Tm)	2.336	1.662	1.458	1.134	1.824	1.799	1.475
	D.R.	1.191	1.470	1.607	1.923	1.384	1.397	1.594

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC4+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC4+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
TOTEMPL	EL(U)	0.518	0.518	0.518	0.518	0.518	0.518	0.518
	EL(Sm)	-0.151	-0.151	-0.151	-0.151	-0.151	-0.151	-0.151
	(t)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
	EL(T)	0.207	0.166	0.154	0.134	0.176	0.174	0.155
	EL(Tm)	0.056	0.016	0.003	-0.017	0.025	0.024	0.004
	D.R.	4.503	15.026	72.653	-13.162	9.400	9.979	54.727
BUSINESS	EL(U)	-1.660	-1.660	-1.660	-1.660	-1.660	-1.660	-1.660
	EL(Sm)	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010
	(t)	(-10.74)	(-10.74)	(-10.74)	(-10.74)	(-10.74)	(-10.74)	(-10.74)
	EL(T)	-0.664	-0.533	-0.493	-0.430	-0.564	-0.559	-0.496
	EL(Tm)	-0.674	-0.543	-0.503	-0.440	-0.574	-0.569	-0.506
	D.R.	0.478	0.473	0.471	0.466	0.474	0.474	0.471
PRIVATE	EL(U)	0.846	0.846	0.846	0.846	0.846	0.846	0.846
	EL(Sm)	0.227	0.227	0.227	0.227	0.227	0.227	0.227
	(t)	(3.68)	(3.68)	(3.68)	(3.68)	(3.68)	(3.68)	(3.68)
	EL(T)	0.338	0.271	0.251	0.219	0.287	0.285	0.253
	EL(Tm)	0.565	0.499	0.478	0.446	0.515	0.512	0.480
	D.R.	-0.102	-0.183	-0.212	-0.263	-0.162	-0.165	-0.210

6. An overview of the application to Canada, 1976

A question of great interest is whether these results for Germany are somehow unique. To answer it we performed a parallel series of tests on a 4-mode intercity flow matrix for Canada. We will outline here the distinguishing features of this application, focusing on significant resemblances or differences from the German case.

6.1 <u>Flow selection and residual impact criteria</u>

The symmetric O-D matrix contains 120 pairs for which intercity passenger travel was observed in Canada: intercity scheduled buses are the fourth and distinguishing mode and represented 2,6 % of the annual market in 1976. As for the German case, we explain the average O-D flow.

In order to make the specification interesting, we decided to consider the greatest possible number of contiguous neighbours, without requiring a minimum distance. This maximum occurs around 320 km: beyond this distance, the number of “neighbours” increases slowly and they cannot reasonably be described as contiguous. We then defined the three analogs of (NORIC-O), (NORIC-D) and (NORIC-OD). But we also decided to define two further residual impact criteria matrices:

<table>
<tr><td rowspan="2">Let r_{tn}</td><td rowspan="2">{</td><td>= 1</td><td>if the flow is associated with a city of population size within 30 % of the population of the city at the origin or destination;</td><td rowspan="2">, (DORIC)</td></tr>
<tr><td>= 0</td><td>otherwise.</td></tr>
</table>

and

<table>
<tr><td rowspan="2">Let r_{tn}</td><td rowspan="2">{</td><td>= 1</td><td>if either the flow has the same destination, or is a neighbour of the origin, or conversely; or the flow is associated with a city of population size within 30 % of the population of the city at origin or destination;</td><td rowspan="2">, (NODORIC)</td></tr>
<tr><td>= 0</td><td>otherwise.</td></tr>
</table>

where clearly the directed order used for (DORIC) is intended to represent potential competition among cities of the same size class as those involved in the current OD pair and (NODORIC) is the union of (NORIC-OD) and (DORIC). Table 7 summarizes the characteristics of the resulting 5 residual impact matrices.

Table 7. Impact Matrices, Canada 1976, 120 O-D pairs

R matrix with distance by car between 0 and 320 km	Number of lines				Characteristics of positive elements per line		
	Total	With positive elements					
		NONE	SOME				
			different	identical	MIN	MAX	MEAN
NORIC-O	120	14	102	4	1	7	2.83
NORIC-D	120	37	83	0	1	10	2.85
NORIC-OD	120	6	102	12	1	12	5.68
DORIC-POP 30 %	120	4	114	2	1	8	4.13
NODORIC-OD + POP 30 %	120	0	120	0	1	19	8.49

6.2 Model specifics

The mode choice model differed slightly from the German model in terms of socioeconomic variables, which comprised

REVE = the geometric mean of per capita average income in cities at the origin and destination;

LANGFR = an index of linguistic similarity at the origin and destination, defined as {100 - |(% French speaking at origin) - (% French speaking at destination)|};

ODREG = a dummy variable equal to 1 when both origin and destination are within the same province, and 0 otherwise.

In addition, the car alternative included the variable

NUITA = number of overnight stays if the car is used

and, for the air mode, frequency is defined as

FREQ1 = {min [air frequency, max (train frequency, bus frequency)]}.

The Generation-Distribution model includes the linguistic pairing index variable LANGFR, in addition to the three other variables. Generally then, the model for Canada differs slightly from the model for Germany. We now summarize the results shown in details in Appendix B.

6.3 Results

Mode Choice. As in the German case, one can see in Table B.1 that there are large gains in allowing for BCT which suggest a logarithmic form for distance – as in the German case – and for speed; however, both frequency – as in the German case – and price should enter linearly.

Generation-distribution. The BCT cannot improve significantly upon a multiplicative form if it is constrained equal. However, if the BCT on the dependent variable is allowed to differ from the BCT on the explanatory variables, large gains in log likelihood occur in both Table B.2 and Table B.3 (comparing Columns 4 and 5) although the numerical value of the BCT remains close to 0. Allowing each variable to have its BCT makes further gains possible in both cases. In particular, the logarithmic form of the utility term U is rejected, as for the German case.

As in the German case, the introduction of autocorrelation with (NORIC-OD) in Table B.2 and (NORIC-O) and (NORIC-D) in Table B.3 yields large gains. However, the proximity parameter π makes contributions only in the latter cases, because π_1 goes to zero (π_2 stays at 1) at the "origin-contiguity", in contrast with the German case where π_1 remained between 0 and 1. The maximum interaction rule applied in defining the contiguity matrix may explain this result: the dense Quebec-Ontario corridor is such that the "tail" is very long: all origins compete.

It should also be noted that the introduction of spatial correlation lowers the elasticity of the modal utility term about 10 % – in contrast with 20 % for the German case.

The additional series of tests shown in Table B.4 probes further the nature of the spatial autocorrelation under the assumption of a multiplicative model. The idea is to compare (NORIC-OD) results with (DORIC) results and then to test for their union (NODORIC). The (DORIC) results show that one gains significantly in introducing through the structure of residuals the notion of competition among cities of the same class size (the log likelihood gain from Column 1 to column 4 is very large) but that one does not gain further (in Column 5) by introducing a proximity parameter. It is also clear that (NODORIC) results are somewhat inferior to (NORIC-OD) results and that the implicit constraint $\rho_1 = \rho_2$ used in Column 9 is very restrictive.

Table B.5 presents QDF results for Canada. One should note that the diversion rates of the two variables that are present in both total and share models (the income variable REVE and the linguistic similarity index LANGFR) are positive, as (35) allows, and very large for the modes that have a small market share.

7. Conclusion

In this paper, we have developed a general approach to the explanation of transport flows that combines into a consistent format the traditional mode choice and generation-distribution models and enriches the overall explanatory power of these models.

A. In terms of specification, the chosen quasi-direct format

> **A.1.** uses in the generation-distribution formula a meaningful index of the attractiveness or utility of the available transportation alternatives.

Frequently, generation-distribution models use a very simplistic specification of the importance of transportation, typically representing its role by a single variable, such as cost or distance by a prevailing mode. In our approach, all price and service levels of all available modes appear in the index because it is defined as the denominator of the logit mode choice model itself. In consequence, a particular transport price or service influences both modal choice and the total amount of trip making. Moreover, it is known from utility theory that, under certain conditions, the natural logarithm of the index has a strict interpretation as the expected maximum utility available over all transportation modes.

> **A.2.** adds to the pair-specific variables currently used to explain each specific origin-destination flow the influence of other alternatives.

The most questionable feature of the specification of generation-distribution models is that they make the flow for a given origin-destination pair depend only upon the transport and socioeconomic conditions of that pair because collinearity arises as soon as the determinants of other opportunities are also used – assuming that one can even select the appropriate subset of relevant other competing or complementary opportunities. Our approach solves both the selection and the collinearity problems by formulating testable hypotheses for the selection of relevant alternatives on the basis of correlation among error terms (normally caused by missing explanatory variables in the models) and weighing the importance of such additional alternatives through a correlation parameter that should generally reduce multicollinearity. This indirect way of introducing “other” variables than “own” variables in the explanation of flows is flexible and adaptable to the specifics of each problem.

B. In terms of calibration of the importance of all variables, our procedures allow the data to determine whether the best mathematical form is that most frequently used or different, for instance

B.1. in the mode choice model we test whether changes at the margin have a constant influence on choice probabilities, and the extent to which, in particular, asymmetric reaction thresholds may be present.

We naturally use as mode choice model the logit model but probe the mathematical form of the utility functions. We reject the linear form. Although generally used, this form incorporates the implicit assumption that modifications in travel conditions have effects that are independent of trip length and uniform across the modes. The values of the estimated Box-Cox transformations indicate that utility functions are nonlinear and imply the existence of asymmetric thresholds in the reaction of travellers. Our flexible specification therefore strongly rejects the popular linear form often accepted without due probing of its behavioural meaning and empirical validity.

B.2. in the generation-distribution piece proper, we allow the interaction to be calibrated and the data to move away from the common multiplicative form.

Although it is natural to expect spatial interactions, such as transport or communication flows, to be determined by a structure in which the influence of each variable depends on the level of that variable and other variables, as happens in a multiplicative model, we fine tune the nature of this interaction, again through the use of Box-Cox transformations. We find that some of the variables should enter multiplicatively, but that others should not and may even enter additively. We find large gains in adjustment to the data even when numerical values obtained for the transformations are not apparently very different form those corresponding to a multiplicative form.

C. In terms of extraction of information from the data, we purge the residual errors from systematic information that they may contain, thus simultaneously obtaining conditions of the randomness, constancy of variance and independence that make our statistical tests more reliable.

Due to the fact that it is impossible to specify a perfect model, it is essential to make a model of the error terms, to extract systematic information that they are expected to contain. The formulation of systematic relationships to account for correlations among observations, or correct the error variance in order to make it constant, is essential not only to account for observed flows as adequately as possible, but also to obtain a model error that satisfies conditions under which unbiased statistics – such as t-statistics – can be obtained. Such models of error terms also influence the estimates obtained for all parameters of the structure. They influence the meaning of the structure, modify the effective patterns of correlation among explanatory variables and qualify the measures of certainty and various tests that are usually performed for any explanatory structure. Our particular emphasis on the spatial nature of information contained in transportation models in particular requires major innovations in existing calibration procedures. We found that such probing had significant impacts on the model parameters, notably on the elasticity of demand, and that the impacts were in the expected direction: it has been shown elsewhere (Picard, Nguyen and Gaudry, 1988) that impedances produced by these models are too high because they do not take into account the input-output constraints that hold for the economy as whole and incorporate a specific competitive spatial structure.

Our approach is therefore rich in terms of specification, flexible in terms of functional form and effective in terms of extraction of information, or adjustment to the data.

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APPENDIX

Table A.1. Generation-Distribution Models, Different Residual Impact Criteria Utility from Variant 3 (Table 2), Germany 1985 (Symmetric Flows)

VARIANT	1	2	3	4	5	6	7	8	9	10
CLASS	LOG	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+
SUBCLASS		AU	AU+PR	AU	AU+PR	AU	AU+PR	AU+PR	AU	AU+PR
INDEPENDENT VARIABLES					ELASTICITY (CONDITIONAL T)					
POPULATION X1	1.37 (14.35)	1.42 (16.81)	1.42 (16.71)	1.44 (16.62)	1.44 (16.42)	1.48 (17.51)	1.50 (17.49)	1.50 (17.27)	1.51 (17.52)	1.49 (16.52)
INCOME X2	1.33 (10.16)	1.55 (9.91)	1.63 (10.02)	1.37 (9.96)	1.35 (9.58)	1.57 (10.13)	1.63 (10.17)	1.55 (9.73)	1.62 (10.26)	1.64 (10.18)
UTILITY X3	0.40 (9.68)	0.33 (7.94)	0.33 (7.89)	0.41 (10.37)	0.41 (10.33)	0.34 (8.17)	0.34 (8.09)	0.34 (8.11)	0.32 (7.85)	0.30 (7.22)
BOX-COX TRANSFORMATIONS					PARAMETER VALUE (UNCONDITIONAL T with PAR=0) <UNCONDITIONAL T with PAR=1>					
DEPENDENT VARIABLE										
LAMBDA Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
INDEPENDENT VARIABLES										
LAMBDA X1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SPATIAL CORRELATION					PARAMETER VALUE (CONDITIONAL T with PAR=0) <CONDITIONAL T with PAR=1>					
O: DIST. (100-180 km)										
RHO (DIST_O)		0.40 (3.82)	0.49 (3.47)			0.39 (3.83)	0.46 (3.37)	0.39 (3.83)		
PI (DIST_O)		1.00	0.61 (1.40) <-0.89>			1.00	0.66 (1.49) <-0.78>	1.00		
D: DIST. (100-180 km)										
RHO (DIST_D)				0.38 (2.63)	0.44 (2.14)	0.36 (2.82)	0.34 (2.73)	0.42 (2.27)		
PI (DIST_D)				1.00	0.67 (0.77) <-0.37>	1.00	1.00	0.66 (0.77) <-0.40>		
O and D: DIST. (100-180 km)										
RHO (DIST_OD)									0.52 (6.31)	0.73 (6.44)
PI (DIST_OD)									1.00	0.40 (1.92) <-2.90>
LOG-LIKELIHOOD	-3057.31	-3047.70	-3046.65	-3053.21	-3052.98	-3043.34	-3042.35	-3043.04	-3040.22	-3032.41
PSEUDO-(L)-R2 (adjusted for D.F.)	0.91	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.93
NUMBER OF PAIRS	286	286	286	286	286	286	286	286	286	286

Table B.1. Linear and Box-Cox Logit Share Models, Canada 1976 (Symmetric Flows)

VARIANT		1	2	3
INDEPENDENT VARIABLE	BETA COEFFICIENT	OWN ELASTICITY (CONDITIONAL t)		
NETWORK		ALTERNATIVE: AIR		
F1/D1	GEN	-1.37 (-5.05)	-1.67 (-5.29)	-0.95 (-7.38)
D1/T1	GEN	0.60 (1.64)	1.29 (5.87)	1.16 (6.24)
DIST1	GEN	-2.71 (-6.81)	-2.22 (-12.63)	-1.22 (-14.45)
FREQU1	GEN	0.29 (5.45)	0.38 (5.56)	0.26 (6.25)
SOCIOECONOMIC				
REVE	SPE	3.33 (1.69)	0.92 (0.79)	1.36 (1.29)
LANGFR	SPE	-0.28 (-0.92)	0.17 (1.10)	0.40 (2.53)
ODREG	SPE	-0.06 (-0.43)	0.11 (1.24)	-0.03 (-0.29)
NETWORK		ALTERNATIVE: RAIL		
F2/D2	GEN	-0.66 (-5.05)	-2.06 (-5.29)	-0.32 (-7.38)
D2/T2	GEN	0.17 (1.64)	1.48 (5.87)	1.41 (6.24)
DIST2	GEN	-3.75 (-6.81)	-3.19 (-12.63)	-1.76 (-14.45)
FREQ2	GEN	0.22 (5.45)	0.50 (5.56)	0.25 (6.25)
SOCIOECONOMIC				
REVE	SPE	2.01 (1.51)	2.05 (1.41)	1.92 (1.57)
LANGFR	SPE	-0.22 (-0.94)	-0.03 (0.40)	-0.19 (0.30)
ODREG	SPE	0.00 (-0.13)	0.05 (1.03)	0.03 (0.17)
NETWORK		ALTERNATIVE: BUS		
F3/D3	GEN	-0.58 (-5.05)	-2.06 (-5.29)	-0.26 (-7.38)
D3/T3	GEN	0.17 (1.64)	1.51 (5.87)	1.44 (6.24)
DIST3	GEN	-4.07 (-6.81)	-3.29 (-12.63)	-1.77 (-14.45)
FREQ3	GEN	0.51 (5.45)	0.58 (5.56)	0.53 (6.25)
SOCIOECONOMIC				
REVE	SPE	0.01 (0.66)	-0.98 (-0.21)	-0.57 (0.30)
LANGFR	SPE	0.71 (2.42)	0.85 (3.94)	0.59 (3.62)
ODREG	SPE	-0.04 (-0.52)	0.06 (1.27)	0.16 (1.51)

VARIANT		1	2	3
INDEPENDENT VARIABLE	BETA COEFFICIENT	OWN ELASTICITY (CONDITIONAL t)		
NETWORK		ALTERNATIVE: CAR		
F4/D4	GEN	-0.20 (-5.05)	-0.89 (-5.29)	-0.14 (-7.38)
D4/T4	GEN	0.06 (1.64)	0.66 (5.87)	0.68 (6.24)
DIST4	GEN	-1.14 (-6.81)	-1.38 (-12.63)	-0.83 (-14.45)
NUITA	SPE	-0.21 (-7.99)	-0.17 (-4.58)	-0.29 (-8.43)
BOX-COX		PARAMETER VALUE (UNCONDITIONAL t with PAR=0) <UNCONDITIONAL t with PAR=1>		
LAMBDA	PRICE	1.00	0.14 (1.99) <-12.20>	1.63 (2.89) <1.12>
	SPEED	1.00	0.14 (1.99) <-12.20>	0.15 (0.61) <-3.52>
	DIST	1.00	0.14 (1.99) <-12.20>	-0.25 (-1.83) <-9.06>
	FREQ	1.00	0.14 (1.99) <-12.20>	0.85 (2.70) <-0.47>
LOG-LIKELIHOOD		-537.74	-494.56	-479.51
R2 (overall)		0.89	0.95	0.95
NUMBER OF PAIRS		120	120	120

Table B.2. Generation-Distribution Models, First Order AR-C-D Process Utility from Variant 3 (Table B.1), Canada 1976 (Symmetric Flows)

VARIANT		1	2	3	4	5	6
CLASS		LOG	LOG+	LOG+	BC1+	BC2+	BC5+
SUBCLASS			AU	AU+PR	AU+PR	AU+PR	AU+PR
INDEPENDENT VARIABLES		ELASTICITY (CONDITIONAL t)					
POPULATION	X1	1.39	1.42	1.42	1.44	1.51	1.54
		(25.45)	(28.04)	(28.36)	(29.98)	(32.66)	(37.50)
REVE	X2	0.83	1.80	1.71	1.89	1.21	1.14
		(1.20)	(2.39)	(2.22)	(2.50)	(1.65)	(2.23)
LANGFR	X3	0.07	0.33	0.35	0.37	0.32	0.22
		(1.20)	(4.08)	(4.16)	(4.42)	(4.69)	(6.69)
UTILITY	X4	0.63	0.57	0.57	0.55	0.66	0.69
		(42.33)	(25.07)	(25.27)	(25.99)	(21.13)	(24.37)
BOX-COX TRANSFORMATIONS		PARAMETER VALUE (UNCONDITIONAL t with PAR=0) <UNCONDITIONAL t with PAR=1>					
DEPENDENT VARIABLE							
LAMBDA	Y	0.00	0.00	0.00	-0.02	0.06	0.06
					(1.39)	(4.02)	(2.40)
					<-63.10>	<-64.36>	<-36.42>
INDEPENDENT VARIABLES							
LAMBDA	X1	0.00	0.00	0.00	-0.02	-0.05	0.16
					(1.39)	(-1.96)	(2.57)
					<-63.10>	<-43.92>	<-13.70>
	X2	0.00	0.00	0.00	-0.02	-0.05	20.00
					(1.39)	(-1.96)	(1.24)
					<-63.10>	<-43.92>	<1.18>
	X3	0.00	0.00	0.00	-0.02	-0.05	-0.15
					(1.39)	(-1.96)	(-0.43)
					<-63.10>	<-43.92>	<-3.31>
	X4	0.00	0.00	0.00	-0.02	-0.05	-0.08
					(1.39)	(-1.96)	(-3.24)
					<-63.10>	<-43.92>	<-44.72>
SPATIAL CORRELATION		PARAMETER VALUE (CONDITIONAL t with PAR=0) <CONDITIONAL t with PAR=1>					
O and D: DIST. < 320 km							
RHO (DIST_OD)			0.75	0.76	0.78	0.77	0.86
			(11.00)	(10.95)	(12.63)	(10.71)	(7.82)
PI (DIST_OD)			1.00	0.84	0.86	0.44	0.00
				(2.72)	(2.76)	(1.37)	(0.00)
				<-0.52>	<-0.45>	<-1.74>	<-37867.5>
LOG-LIKELIHOOD		-1318.24	-1294.58	-1294.48	-1293.94	-1279.71	-1262.98
PSEUDO-(L)-R2 (adjusted for D.F.)		0.99	0.99	0.99	0.99	0.99	0.99
NUMBER OF PAIRS		120	120	120	120	120	120

Table B.3. Generation-Distribution Models, Second Order AR-C-D Process Utility from Variant 3 (Table B.1), Canada 1976 (Symmetric Flows)

VARIANT CLASS SUBCLASS		1 LOG+	2 LOG+ AU	3 LOG+ AU+PR	4 BC1+ AU+PR	5 BC2+ AU+PR	6 BC5+ AU+PR
INDEPENDENT VARIABLES		ELASTICITY (CONDITIONAL t)					
POPULATION	X1	1.39 (25.45)	1.41 (25.61)	1.43 (29.06)	1.46 (31.40)	1.50 (37.05)	1.50 (30.01)
REVE	X2	0.83 (1.20)	1.32 (1.56)	0.94 (1.28)	1.35 (1.95)	0.21 (0.31)	0.31 (0.58)
LANGFR	X3	0.07 (1.20)	0.24 (2.39)	0.29 (3.76)	0.31 (4.24)	0.25 (4.22)	0.20 (5.94)
UTILITY	X4	0.63 (42.33)	0.60 (21.25)	0.54 (18.76)	0.51 (19.10)	0.66 (22.20)	0.67 (22.83)
BOX-COX TRANSFORMATIONS		PARAMETER VALUE (UNCONDITIONAL t with PAR=0) <UNCONDITIONAL t with PAR=1>					
DEPENDENT VARIABLE							
LAMBDA	Y	0.00	0.00	0.00	0.03 (1.57) <-51.44>	0.06 (3.21) <-51.72>	0.07 (2.07) <-29.05>
INDEPENDENT VARIABLES							
LAMBDA	X1	0.00	0.00	0.00	0.03 (1.57) <-51.44>	-0.06 (-2.53) <-48.52>	0.18 (2.49) <-11.22>
	X2	0.00	0.00	0.00	0.03 (1.57) <-51.44>	-0.06 (-2.53) <-48.52>	6.55 (0.14) <0.12>
	X3	0.00	0.00	0.00	0.03 (1.57) <-51.44>	-0.06 (-2.53) <-48.52>	-0.25 (-0.42) <-2.12>
	X4	0.00	0.00	0.00	0.03 (1.57) <-51.44>	-0.06 (-2.53) <-48.52>	-0.07 (-2.75) <-44.08>
SPATIAL CORRELATION		PARAMETER VALUE (CONDITIONAL t with PAR=0) <CONDITIONAL t with PAR=1>					
O: DIST. < 320 km							
RHO (DIST_O)			0.42 (3.38)	0.59 (6.15)	0.63 (7.25)	0.52 (5.01)	0.57 (5.72)
PI (DIST_O)			1.00	0.00 (0.00) <-22.21>	0.01 (0.23) <-15.96>	0.00 (0.00) <-22.57>	0.00 (0.00) <-7.19>
D: DIST. < 320 km							
RHO (DIST_D)			0.32 (1.87)	0.21 (1.31)	0.22 (1.64)	0.24 (1.40)	0.25 (1.84)
PI (DIST_D)			1.00	1.00 (0.64) <0.00>	1.00 (0.69) <0.00>	1.00 (0.67) <0.00>	1.00 (0.88) <0.00>
LOG-LIKELIHOOD		-1318.24	-1309.49	-1301.11	-1300.22	-1287.21	-1272.27
PSEUDO-(L)-R2 (adjusted for D.F.)		0.99	0.99	0.99	0.99	0.99	0.99
NUMBER OF PAIRS		120	120	120	120	120	120

Table B.4. Generation-Distribution Models, Different Residual Impact Criteria Utility from Variant 3 (Table B.1), Canada 1976 (Symmetric Flows)

VARIANT	1	2	3	4	5	6	7	8	9	10
CLASS	LOG	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+	LOG+
SUBCLASS		AU	AU+PR	AU	AU+PR	AU	AU+PR	AU+PR	AU	AU+PR
INDEPENDENT VARIABLES					ELASTICITY (CONDITIONAL t)					
POPULATION X1	1.39	1.42	1.42	1.42	1.42	1.46	1.47	1.46	1.47	1.47
	(25.45)	(28.04)	(28.36)	(15.67)	(15.57)	(21.51)	(21.80)	(21.51)	(17.65)	(17.63)
REVE X2	0.83	1.80	1.71	0.30	0.30	1.72	1.52	1.72	0.48	0.48
	(1.20)	(2.39)	(2.22)	(0.39)	(0.39)	(2.33)	(1.92)	(2.31)	(0.61)	(0.59)
LANGFR X3	0.07	0.33	0.35	0.14	0.14	0.31	0.32	0.31	0.24	0.24
	(1.20)	(4.08)	(4.16)	(2.34)	(2.26)	(5.06)	(5.13)	(4.78)	(3.65)	(3.56)
UTILITY X4	0.63	0.57	0.57	0.60	0.60	0.56	0.55	0.56	0.58	0.58
	(42.33)	(25.07)	(25.27)	(39.11)	(38.76)	(29.94)	(29.95)	(27.94)	(27.26)	(26.96)
BOX-COX TRANSFORMATIONS					PARAMETER VALUE (UNCONDITIONAL t with PAR=0) <UNCONDITIONAL t with PAR=1>					
DEPENDENT VARIABLE										
LAMBDA Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
INDEPENDENT VARIABLES										
LAMBDA X1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SPATIAL CORRELATION					PARAMETER VALUE (CONDITIONAL t with PAR=0) <CONDITIONAL t with PAR=1>					
O and D: DIST. < 320 km										
RHO (DIST_OD)		0.75	0.76			0.59	0.60	0.59		
		(11.00)	(10.95)			(7.65)	(7.89)	(7.30)		
PI (DIST_OD)		1.00	0.84			1.00	0.66	1.00		
			(2.72)				(1.83)			
			<-0.52>				<-0.95>			
O and D: POP. (30%)										
RHO (POP_OD)				0.52	0.52	0.38	0.37	0.38		
				(3.41)	(3.14)	(4.21)	(4.13)	(3.86)		
PI (POP_OD)				1.00	1.00	1.00	1.00	1.00		
					(2.02)			(1.81)		
					<0.00>			<0.00>		
O and D: DIST. < 320 km and POP. (30%)										
RHO (DIST_POP)									0.80	0.80
									(7.07)	(6.25)
PI (DIST_POP)									1.00	1.00
										(2.71)
										<0.00>
LOG-LIKELIHOOD	-1318.24	-1294.58	-1294.48	-1307.62	-1307.62	-1283.89	-1283.64	-1283.89	-1301.10	-1301.10
PSEUDO-(L)-R2 (adjusted for D.F.)	0.99	0.99	0.99	0.99	0.99	0.99	0.92	0.92	0.99	0.99
NUMBER OF PAIRS	120	120	120	120	120	120	120	120	120	120

Table B.5. Share (Sm), Total (T0 and Modal (Tm) Elasticities; Diversion Rates (D.R.) Models from Table B.2 and Table B.3, Canada 1976 (Symmetric Flows)

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC5+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC5+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
LEVEL								
POPULATION	EL(T,X)	1.389	1.419	1.425	1.540	1.414	1.430	1.498
	(t)	(25.45)	(28.04)	(28.36)	(37.50)	(25.61)	(29.06)	(30.01)
REVE	EL(T,X)	0.832	1.799	1.706	1.138	1.320	0.940	0.308
	(t)	(1.20)	(2.39)	(2.22)	(2.23)	(1.56)	(1.28)	(0.58)
LANGFR	EL(T,X)	0.066	0.334	0.348	0.216	0.239	0.287	0.195
	(t)	(1.20)	(4.08)	(4.16)	(6.69)	(2.39)	(3.76)	(5.94)
UTILITY	EL(T,U)	0.634	0.571	0.569	0.687	0.596	0.540	0.674
	(t)	(42.33)	(25.07)	(25.27)	(24.37)	(21.25)	(18.76)	(22.83)
SHARE								
ALTERNATIVE (MEAN = 0.365): AIR								
F1/D1	EL(U)	-0.547	-0.547	-0.547	-0.547	-0.547	-0.547	-0.547
	EL(Sm)	-0.952	-0.952	-0.952	-0.952	-0.952	-0.952	-0.952
	(t)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)
	EL(T)	-0.347	-0.313	-0.311	-0.376	-0.326	-0.296	-0.369
	EL(Tm)	-1.299	-1.264	-1.263	-1.328	-1.278	-1.247	-1.321
	D.R.	-0.268	-0.322	-0.324	-0.224	-0.300	-0.350	-0.234
D1/T1	EL(U)	0.670	0.670	0.670	0.670	0.670	0.670	0.670
	EL(Sm)	1.164	1.164	1.164	1.164	1.164	1.164	1.164
	(t)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)
	EL(T)	0.425	0.382	0.381	0.460	0.399	0.362	0.451
	EL(Tm)	1.589	1.547	1.546	1.625	1.564	1.526	1.616
	D.R.	-0.268	-0.322	-0.324	-0.224	-0.300	-0.350	-0.234
DIST1	EL(U)	-0.702	-0.702	-0.702	-0.702	-0.702	-0.702	-0.702
	EL(Sm)	-1.221	-1.221	-1.221	-1.221	-1.221	-1.221	-1.221
	(t)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)
	EL(T)	-0.445	-0.401	-0.399	-0.482	-0.418	-0.379	-0.473
	EL(Tm)	-1.666	-1.622	-1.620	-1.703	-1.639	-1.600	-1.694
	D.R.	-0.268	-0.322	-0.324	-0.224	-0.300	-0.350	-0.234
FREQU1	EL(U)	0.147	0.147	0.147	0.147	0.147	0.147	0.147
	EL(Sm)	0.256	0.256	0.256	0.256	0.256	0.256	0.256
	(t)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)
	EL(T)	0.093	0.084	0.084	0.101	0.088	0.079	0.099
	EL(Tm)	0.349	0.340	0.340	0.357	0.343	0.335	0.355
	D.R.	-0.268	-0.322	-0.324	-0.224	-0.300	-0.350	-0.234
REVE	EL(U)	1.043	1.043	1.043	1.043	1.043	1.043	1.043
	EL(Sm)	1.362	1.362	1.362	1.362	1.362	1.362	1.362
	(t)	(1.29)	(1.29)	(1.29)	(1.29)	(1.29)	(1.29)	(1.29)
	EL(T)	1.493	2.395	2.299	1.855	1.942	1.503	1.011
	EL(Tm)	2.855	3.757	3.662	3.217	3.304	2.865	2.373
	D.R.	0.433	0.747	0.721	0.580	0.611	0.438	0.168

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC5+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC5+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
LANGFR	EL(U)	0.274	0.274	0.274	0.274	0.274	0.274	0.274
	EL(Sm)	0.396	0.396	0.396	0.396	0.396	0.396	0.396
	(t)	(2.53)	(2.53)	(2.53)	(2.53)	(2.53)	(2.53)	(2.53)
	EL(T)	0.240	0.491	0.504	0.405	0.403	0.435	0.380
	EL(Tm)	0.636	0.886	0.900	0.800	0.798	0.831	0.776
	D.R.	0.035	0.517	0.536	0.385	0.382	0.435	0.343
ODREG	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
	EL(Sm)	-0.029	-0.029	-0.029	-0.029	-0.029	-0.029	-0.029
	(t)	(-0.29)	(-0.29)	(-0.29)	(-0.29)	(-0.29)	(-0.29)	(-0.29)
	EL(T)	-0.005	-0.005	-0.005	-0.006	-0.005	-0.004	-0.005
	EL(Tm)	-0.034	-0.033	-0.033	-0.034	-0.034	-0.033	-0.034
	D.R.	-0.583	-0.618	-0.619	-0.553	-0.604	-0.636	-0.560

ALTERNATIVE (MEAN = 0.041): RAIL

F2/D2	EL(U)	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017
	EL(Sm)	-0.316	-0.316	-0.316	-0.316	-0.316	-0.316	-0.316
	(t)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)
	EL(T)	-0.011	-0.010	-0.010	-0.012	-0.010	-0.009	-0.011
	EL(Tm)	-0.327	-0.326	-0.326	-0.328	-0.326	-0.325	-0.328
	D.R.	-0.197	-0.274	-0.277	-0.132	-0.244	-0.313	-0.148
D2/T2	EL(U)	0.075	0.075	0.075	0.075	0.075	0.075	0.075
	EL(Sm)	1.407	1.407	1.407	1.407	1.407	1.407	1.407
	(t)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)
	EL(T)	0.048	0.043	0.043	0.052	0.045	0.041	0.051
	EL(Tm)	1.455	1.450	1.450	1.459	1.452	1.448	1.458
	D.R.	-0.197	-0.274	-0.277	-0.132	-0.244	-0.313	-0.148
DIST2	EL(U)	-0.094	-0.094	-0.094	-0.094	-0.094	-0.094	-0.094
	EL(Sm)	-1.760	-1.760	-1.760	-1.760	-1.760	-1.760	-1.760
	(t)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)
	EL(T)	-0.060	-0.054	-0.054	-0.065	-0.056	-0.051	-0.064
	EL(Tm)	-1.819	-1.813	-1.813	-1.824	-1.816	-1.811	-1.823
	D.R.	-0.197	-0.274	-0.277	-0.132	-0.244	-0.313	-0.148
FREQ2	EL(U)	0.014	0.014	0.014	0.014	0.014	0.014	0.014
	EL(Sm)	0.253	0.253	0.253	0.253	0.253	0.253	0.253
	(t)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)
	EL(T)	0.009	0.008	0.008	0.009	0.008	0.007	0.009
	EL(Tm)	0.262	0.261	0.261	0.263	0.262	0.261	0.263
	D.R.	-0.197	-0.274	-0.277	-0.132	-0.244	-0.313	-0.148
REVE	EL(U)	1.043	1.043	1.043	1.043	1.043	1.043	1.043
	EL(Sm)	1.925	1.925	1.925	1.925	1.925	1.925	1.925
	(t)	(1.57)	(1.57)	(1.57)	(1.57)	(1.57)	(1.57)	(1.57)
	EL(T)	1.493	2.395	2.299	1.855	1.942	1.503	1.011
	EL(Tm)	3.418	4.319	4.224	3.779	3.866	3.428	2.936
	D.R.	9.679	12.551	12.306	10.995	11.275	9.719	7.418

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC5+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC5+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
LANGFR	EL(U)	0.274	0.274	0.274	0.274	0.274	0.274	0.274
	EL(Sm)	-0.189	-0.189	-0.189	-0.189	-0.189	-0.189	-0.189
	(t)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)
	EL(T)	0.240	0.491	0.504	0.405	0.403	0.435	0.380
	EL(Tm)	0.051	0.302	0.315	0.215	0.214	0.246	0.191
	D.R.	114.154	38.764	38.110	44.889	45.087	42.218	47.649
ODREG	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
	EL(Sm)	0.027	0.027	0.027	0.027	0.027	0.027	0.027
	(t)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)
	EL(T)	-0.005	-0.005	-0.005	-0.006	-0.005	-0.004	-0.005
	EL(Tm)	0.022	0.022	0.022	0.021	0.022	0.022	0.021
	D.R.	-6.838	-6.136	-6.114	-7.454	-6.410	-5.802	-7.301

ALTERNATIVE (MEAN = 0.026): BUS

F3/D3	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
	EL(Sm)	-0.256	-0.256	-0.256	-0.256	-0.256	-0.256	-0.256
	(t)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)
	EL(T)	-0.005	-0.004	-0.004	-0.005	-0.005	-0.004	-0.005
	EL(Tm)	-0.261	-0.261	-0.261	-0.261	-0.261	-0.260	-0.261
	D.R.	-0.282	-0.352	-0.354	-0.223	-0.324	-0.387	-0.237
D3/T3	EL(U)	0.043	0.043	0.043	0.043	0.043	0.043	0.043
	EL(Sm)	1.435	1.435	1.435	1.435	1.435	1.435	1.435
	(t)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)
	EL(T)	0.027	0.025	0.025	0.030	0.026	0.023	0.029
	EL(Tm)	1.463	1.460	1.460	1.465	1.461	1.459	1.464
	D.R.	-0.282	-0.352	-0.354	-0.223	-0.324	-0.387	-0.237
DIST3	EL(U)	-0.053	-0.053	-0.053	-0.053	-0.053	-0.053	-0.053
	EL(Sm)	-1.773	-1.773	-1.773	-1.773	-1.773	-1.773	-1.773
	(t)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)
	EL(T)	-0.034	-0.031	-0.030	-0.037	-0.032	-0.029	-0.036
	EL(Tm)	-1.807	-1.804	-1.804	-1.810	-1.805	-1.802	-1.809
	D.R.	-0.282	-0.352	-0.354	-0.223	-0.324	-0.387	-0.237
FREQ3	EL(U)	0.016	0.016	0.016	0.016	0.016	0.016	0.016
	EL(Sm)	0.530	0.530	0.530	0.530	0.530	0.530	0.530
	(t)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)
	EL(T)	0.010	0.009	0.009	0.011	0.010	0.009	0.011
	EL(Tm)	0.540	0.539	0.539	0.541	0.539	0.539	0.541
	D.R.	-0.282	-0.352	-0.354	-0.223	-0.324	-0.387	-0.237
REVE	EL(U)	1.043	1.043	1.043	1.043	1.043	1.043	1.043
	EL(Sm)	-0.571	-0.571	-0.571	-0.571	-0.571	-0.571	-0.571
	(t)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)
	EL(T)	1.493	2.395	2.299	1.855	1.942	1.503	1.011
	EL(Tm)	0.922	1.823	1.728	1.283	1.370	0.932	0.440
	D.R.	60.975	49.255	49.914	54.300	53.217	60.721	86.980

VARIANT		1	2	3	4	5	6	7
LEVEL		LOG	LOG+ AU1	LOG+ AU1+PR	BC5+ AU1+PR	LOG+ AU1+2	LOG+ AU1+2+PR	BC5+ AU1+2+PR
SHARE		VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3	VARIANT 3
LANGFR	EL(U)	0.274	0.274	0.274	0.274	0.274	0.274	0.274
	EL(Sm)	0.593	0.593	0.593	0.593	0.593	0.593	0.593
	(t)	(3.62)	(3.62)	(3.62)	(3.62)	(3.62)	(3.62)	(3.62)
	EL(T)	0.240	0.491	0.504	0.405	0.403	0.435	0.380
	EL(Tm)	0.833	1.084	1.097	0.997	0.995	1.028	0.973
	D.R.	10.026	16.328	16.585	14.520	14.475	15.198	13.945
ODREG	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
	EL(Sm)	0.157	0.157	0.157	0.157	0.157	0.157	0.157
	(t)	(1.51)	(1.51)	(1.51)	(1.51)	(1.51)	(1.51)	(1.51)
	EL(T)	-0.005	-0.005	-0.005	-0.006	-0.005	-0.004	-0.005
	EL(Tm)	0.152	0.153	0.153	0.152	0.152	0.153	0.152
	D.R.	-2.297	-2.164	-2.160	-2.409	-2.217	-2.099	-2.382
ALTERNATIVE (MEAN = 0.568): CAR								
F4/D4	EL(U)	-0.175	-0.175	-0.175	-0.175	-0.175	-0.175	-0.175
	EL(Sm)	-0.140	-0.140	-0.140	-0.140	-0.140	-0.140	-0.140
	(t)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)
	EL(T)	-0.111	-0.100	-0.099	-0.120	-0.104	-0.094	-0.118
	EL(Tm)	-0.251	-0.240	-0.240	-0.260	-0.244	-0.235	-0.258
	D.R.	-0.223	-0.268	-0.270	-0.188	-0.250	-0.292	-0.197
D4/T4	EL(U)	0.846	0.846	0.846	0.846	0.846	0.846	0.846
	EL(Sm)	0.679	0.679	0.679	0.679	0.679	0.679	0.679
	(t)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)
	EL(T)	0.537	0.483	0.482	0.582	0.505	0.457	0.571
	EL(Tm)	1.216	1.163	1.161	1.261	1.184	1.137	1.250
	D.R.	-0.223	-0.268	-0.270	-0.188	-0.250	-0.292	-0.197
DIST4	EL(U)	-1.033	-1.033	-1.033	-1.033	-1.033	-1.033	-1.033
	EL(Sm)	-0.829	-0.829	-0.829	-0.829	-0.829	-0.829	-0.829
	(t)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)
	EL(T)	-0.655	-0.590	-0.588	-0.710	-0.616	-0.558	-0.696
	EL(Tm)	-1.484	-1.419	-1.417	-1.539	-1.445	-1.387	-1.526
	D.R.	-0.223	-0.268	-0.270	-0.188	-0.250	-0.292	-0.197
NUITA	EL(U)	-0.360	-0.360	-0.360	-0.360	-0.360	-0.360	-0.360
	EL(Sm)	-0.289	-0.289	-0.289	-0.289	-0.289	-0.289	-0.289
	(t)	(-8.43)	(-8.43)	(-8.43)	(-8.43)	(-8.43)	(-8.43)	(-8.43)
	EL(T)	-0.228	-0.206	-0.205	-0.247	-0.215	-0.194	-0.243
	EL(Tm)	-0.517	-0.495	-0.494	-0.536	-0.504	-0.484	-0.532
	D.R.	-0.223	-0.268	-0.270	-0.188	-0.250	-0.292	-0.197